AN INVESTIGATION OF SNAP ROLL IN SUBMARINES

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by

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ABSTRACT

Snap roll is a phenomenon which has plagued the submarine designer since the advent of the high speed submarine. A description of snap roll and its possible causes is presented in this thesis. A computer program is developed which simulates a submarine in surge, sway, yaw and roll. It is verified that snap roll is very sensitive to the metacentric height. Additionally, it is seen that reducing the sail size does not have as large an effect on reducing the roll angle as anticipated. Rudder sequencing and speed reduction are two other methods examined. The results of the four investigations are presented in graphic form. The conclusions include a justification for the use of the simulation computer model as a design tool.

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D.J.L.

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NOMENCLATURE

SYMBOL	DEFINITION
CB	Center of buoyancy.
CG	Center of gravity.
D	Drag force.
I_i	Moment of inertia about the i axis.
I_{jk}	Product of inertia about jk axis.
K	Hydrodynamic moment about x axis (rolling moment).
K'_i	Non-dimensional coefficient used in representing K as a function of i.
ℓ	Overall ship length.
L	Lift force.
m	Ship's mass.
N	Hydrodynamic moment component about z axis (yawing moment).
N'_i	Non-dimensional coefficient used in representing N as a function of i.
p	Angular velocity about the x axis.

$\dot{p}$	Angular acceleration about the x axis.
q	Angular velocity about the y axis.
$\dot{q}$	Angular acceleration about the y axis.
r	Angular velocity about the z axis.
$\dot{r}$	Angular acceleration about the z axis.
U	Velocity of the origin of the body axes relative to the fluid.
u	U velocity component in the x direction.
$\dot{u}$	Acceleration component in the x direction.
u_o	Initial steady value of U in the x direction.
v	U velocity component in the y direction.
$\dot{v}$	Acceleration component in the y direction.
w	U velocity component in the z direction.
$\dot{w}$	Acceleration component in the z direction.
x_B	The x coordinate of the center of buoyancy.
x_G	The x coordinate of the center of gravity.

x_s	The x coordinate of the center of pressure of the sail.
X	Hydrodynamic force along x axis (axial force).
X'_i	Non-dimensional coefficient used in representing X as a function of i.
y_B	The y coordinate of the center of buoyancy.
y_G	The y coordinate of the center of gravity.
y_s	The y coordinate of the center of pressure of the sail.
Y	Hydrodynamic force along y axis (lateral force).
Y'_i	Non-dimensional coefficient used in representing Y as a function of i.
z_B	The z coordinate of the center of buoyancy.
z_G	The z coordinate of the center of gravity.
z_s	The z coordinate of the center of pressure of the sail.
β	Angle of drift.
δ_b	Bowplane or sailplane deflection.

δ_r	Rudder deflection.
δ_s	Sternplane deflection.
η	Ratio u_o/u.
θ	Angle of pitch.
ψ	Angle of yaw.
φ	Angle of roll.
a_i, b_i, c_i	Constants used to represent the propeller thrust in the axial equation.

CHAPTER I - INTRODUCTION

I.1 BACKGROUND

Since the advent of the high speed submarine in the late fifties, the submarine designer has been confronted with new design problems in the area of ship control. One of these problems, which is still with us two decades later, is snap roll. Part of the reason for the perpetuation of this problem has been a general lack of adequate data and information concerning this phenomenon. Consequently, appropriate design criteria could not be developed and the problem of snap roll has continued. Naval architects and ship designers need adequate alternatives and trade-offs to provide good designs. This is the motivation for this thesis.

A computer model will be developed in Chapter II which simulates a submarine in roll, yaw, surge and sway. This model can be used by naval architects as a design tool in performing trade-off studies or as a method of establishing design criteria for a particular design. The alternatives for reducing snap roll, which are investigated in Chapter III using this simulation model, were based on the immediate needs of the design community and the time and monetary constraints normally placed on this type of effort. It was felt that these alternatives should not increase the com-

plexity of present designs or alter present design practices, in order that they may be put to immediate use. Chapter IV presents the conclusions and recommendations of this thesis. In the appendix will be found a listing of the computer model, a flow chart and a sample output.

I.2 WHAT IS SNAP ROLL?

An excellent description of snap roll was presented by Griffin, et al (1)[1]:

> "At 20 sec after full rudder, the initial transients have died out, and the lateral states begin to follow the trim values which correspond to the decreasing forward speed. Eventually, the forward velocity reaches its trim value, and r, q, v, w and φ are fixed at their respective trim values. For r, q, v, and w, the difference between the plateau value and the later trim value is not large; however, for roll, it can differ by more than 13 deg. This is snap roll."

Simply stated, snap roll is the maximum roll amplitude that results from a turning maneuver. The large roll angle,

[1] Numbers in parentheses indicate references listed at the end of this thesis

which develops early in the turn, constitutes the "snap." The large peak value of the roll angle is the submarine tending towards its steady state turning value for the initially high forward speed. This usually occurs about 20 seconds into the turn. As the submarine enters the turn and during the initial phase of the turn, there is a large side slip velocity. This side slip velocity probably accounts for a major portion of the maximum roll angle. The subsequent reduction of the large roll angle is primarily due to speed loss. The high drag force developed during the turn maneuver dramatically reduces the forward velocity component. The steady state forward speed can be expected to be 1/4 to 1/3 of the initial speed.

There is still conjecture amongst hydrodynamicists as to the theory and physics involved in explaining this phenomenon. It is generally agreed that snap roll primarily depends on the turning rate, the amount of rudder deflection, the fairwater (sail) size and location, and the ship speed. A large turning rate and/or rudder deflection produces a large lateral velocity. Consequently, this large v acting on the sail produces a large roll moment. Additionally, it has been proposed that the starting lift exceeds the steady lift due to the dominating effect of the trailing vortex sheet downwash in the steady case (2). This may be due to the starting vortex formed and shed by

the sail. However, there is no substantiating evidence for this.

CHAPTER II - THE SIMULATION MODEL AND COMPUTER PROGRAM

II.1 THE FOUR DEGREE OF FREEDOM MODEL

There exists at least three computer programs which simulate a submarine in a maneuver. However, none of these programs are available for general use, primarily due to proprietary considerations. Therefore, in order to investigate snap roll in submarines, it was necessary to develop another simulation model. Unlike the other models, however, this model involves the use of four degrees of freedom vice six degrees of freedom equations. The four coupled equations considered here are the X, Y, N and K equations. The justification for the use of only four equations of motion is that in a turn, depth excursions are small and the Z and M equations may be decoupled. The depth change which occurs during a turn, called "squat". normally can be easily remedied with a small deflection of the diving planes and/or sternplanes. These small excursions have a negligble effect on snap roll. This is evidenced by the fact that a four degree of freedom simulation program provides valid results for a turning maneuver. This will be seen from the results presented later in this chapter and in Chapter III. The advantages afforded by a four degree of freedom program, in addition to providing valid results, are:

- Reduced computation time.
- The estimation or measurement of fewer hydrodynamic coefficients.

Of course, such a model, necessarily restricts one's consideration to only maneuvers involving surge, sway, roll and yaw.

II.2 THE COORDINATE SYSTEM AND EQUATIONS

The coordinate system chosen for use in this model was based on reference (3). This reference gives the standard equations of motion for submarines used by the U.S. Navy. The coordinate system is illustrated in figure 2-1. It should be noted that the origin is centered at the center of gravity. A better choice may have been to use the center of buoyancy; however, to avoid a rewriting of already valid equations, the reference (3) system was used. The center of buoyancy of the basic hull for a submarine design is necessarily the geometric center of the hull. Therefore, a transverse velocity (or acceleration) would not produce a roll moment, caused by the basic hull form and this would reduce the number of necessary calculations.

The equations used in the development of this simulation model were taken directly from reference (3) and are listed in Appendix B. There are no second order or higher

Coordinate System

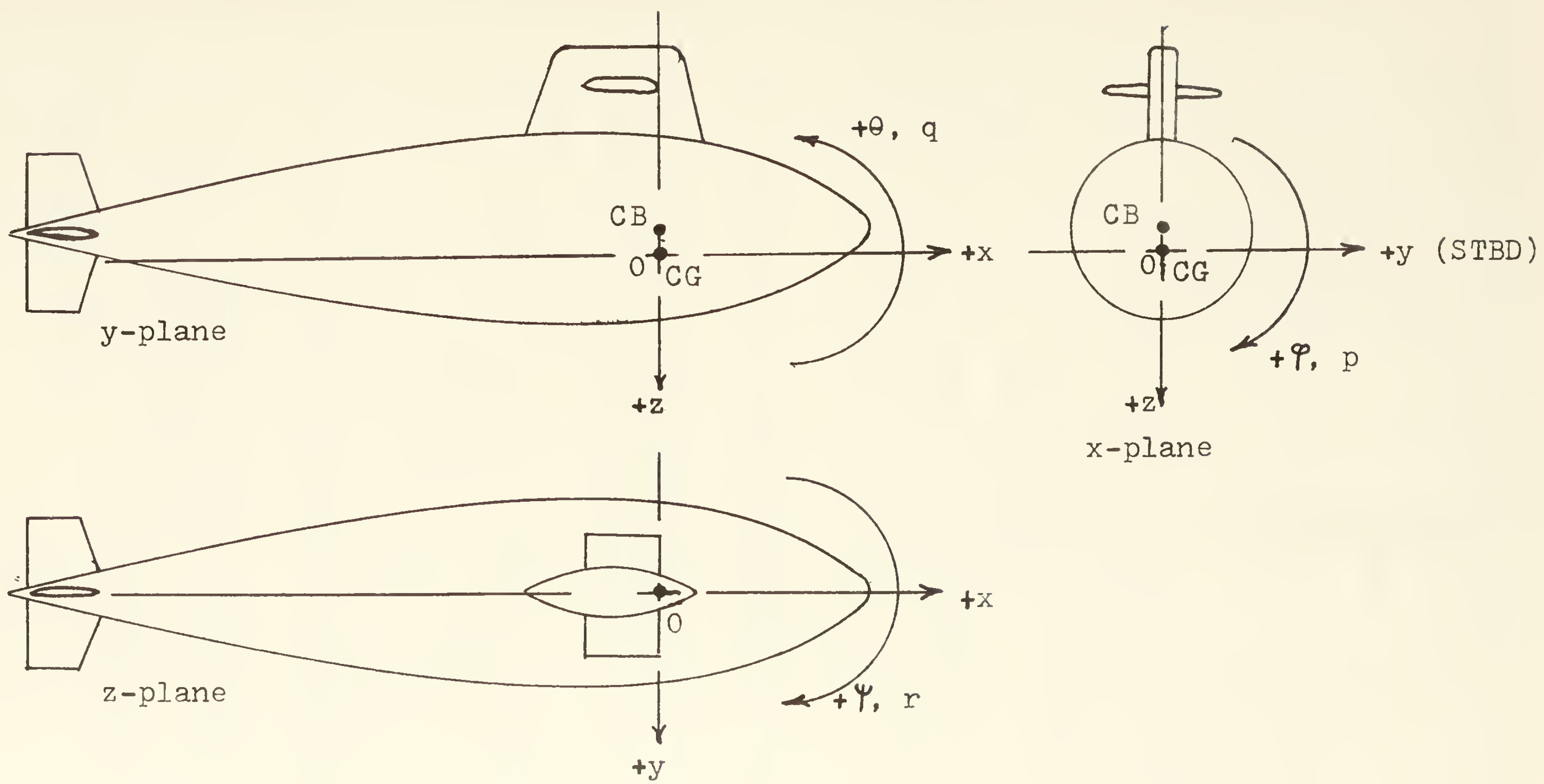

Figure 2-1

order acceleration terms involved in these equations. This is due to the lack of any significant interaction of the viscous and inertia forces. This allows the use of potential theory which gives adequate values for the hydrodynamic forces (4). Acceleration and velocity interaction terms are also assumed negligibly small for the same reason stated above. It should be noted that the equations remain non-linear even without these second order acceleration terms, due to the higher order velocity terms.

The equations are applied to a submarine in a turning maneuver under the following assumptions and restrictions:

1. The submarine is neutrally buoyant, i.e. $W - B = 0$.
2. The submarine is at least three hull diameters below the free surface, i.e. fully submerged.
3. The z component of the velocity is zero.
4. There is no initial list, i.e. $\varphi_o = 0$.
5. There is zero trim, i.e. $\theta = q = \dot{q} = 0$.
6. The origin is at the center of gravity, i.e. $x_G = y_G = z_G = 0$.
7. Only the moments of inertia along the principal axes are non-zero, i.e. $I_{xy} = I_{yz} = I_{xz} = 0$. This is partly a consequence of #6.

8. The coefficients and equations used are in non-dimensional form.

9. The control surface deflections are initially zero, with only the rudder angle being non-zero for $t>0$.

II.3 THE SOLUTION OF THE EQUATIONS OF MOTION

The simplest method of solving for the roll angle induced by a turning maneuver is to solve for u(t) from the non-linear X equation; then, to use this u in solving the linear Y, N and K equations. This will provide comparatively accurate values for the roll angle for relatively small deflections and for small forward speed losses. However, we are interested in high speed and tight turns, where snap roll is most prominent. The vehicle motions in this type of turning maneuver are outside the valid linear range. Therefore, the non-linear equations of motion are used.

The general method used for solving the four non-linear equations is an iterative technique. The specific technique employed in this thesis was adapted from the method used in reference (5). The four equations are each written in terms of the accelerations $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$. They take the form:

X: $(m' - X'_{\dot{u}})\,\dot{u} - y_G\dot{r} = f_1\,(u, v, r, p, \delta_r)$

Y: $(m' - Y'_{\dot{v}})\,\dot{v} - (m'z_G + Y'_{\dot{p}}\,l)\,\dot{p} +$

$(m'x_G - Y'_{\dot{r}}\,l) = f_2\,(u, v, r, p, \delta_r)$

N: $(I'_z - N'_{\dot{r}})\,\dot{r} - (I'_{zx} + N'_{\dot{p}})\,\dot{p} +$

$(m'x_G/l^2 - N'_{\dot{v}}/l)\,\dot{v} - (m'y_G/l^2)\,\dot{u} =$

$f_3\,(u, v, r, p, \delta_r)$

K: $(I'_x - K'_{\dot{p}})\,\dot{p} - (I'_{xz} + K'_{\dot{r}})\,\dot{r} -$

$(m'z_G/l^2 + K'_{\dot{v}}/l)\,\dot{v} = f_4\,(u, v, r, p, \delta_r).$

The prime indicates a non-dimensional value.

Expressions for the accelerations $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$ are derived by employing matrix methods. Based on the initial conditions u_o, v_o, etc., starting values for $\dot{u}$, $\dot{v}$, $\dot{r}$ and $\dot{p}$ are obtained for $t = 0$. Values for u, v, r and p are calculated from these starting values employing the Taylor expansions,

$$u(t + \Delta t) \simeq u(t) + \Delta t \cdot \dot{u}(t)$$
$$v(t + \Delta t) \simeq v(t) + \Delta t \cdot \dot{v}(t)$$
$$r(t + \Delta t) \simeq r(t) + \Delta t \cdot \dot{r}(t)$$
$$\varphi(t + \Delta t) \simeq \varphi(t) + \Delta t \cdot p(t) + ((\Delta t)^2) \cdot \dot{p}(t).$$

These new velocity values are then used to update the pre-

viously calculated accelerations. This indicates the iterative nature of this integration technique. The value of δ_r and t are updated after each iteration. The equations are integrated over the time span specified. The size of Δt is arbitrary; however, the smaller the Δt, the greater the accuracy obtained from this technique. The accuracy desired should be tempered by the fact that small Δt's require rather large amounts of computer time. It was found that Δt's on the order of one half to one second provided very satisfactory results, while not using excessive amounts of computer time.

II.4 COMPUTER SIMULATION MODEL VALIDITY

A large number of computer runs were executed for various submarines, under various conditions. It was intended to prove the validity of the developed computerized simulation model using hydrodynamic coefficients taken from towing tank model tests. The results from one of these computer runs is presented in figure 2-2. As can be seen, in general, the model accurately predicts the submarine's response in roll; however, the predicted snap roll angle is too large. This can be attributed to a number of causes. One is the possibility of scale effects, since the coefficients used in the program are not the full scale hydrodynamic coefficients. Secondly, the metacentric height used was that specified in publications. The actual GM

Full Scale Trials and Simulation Program

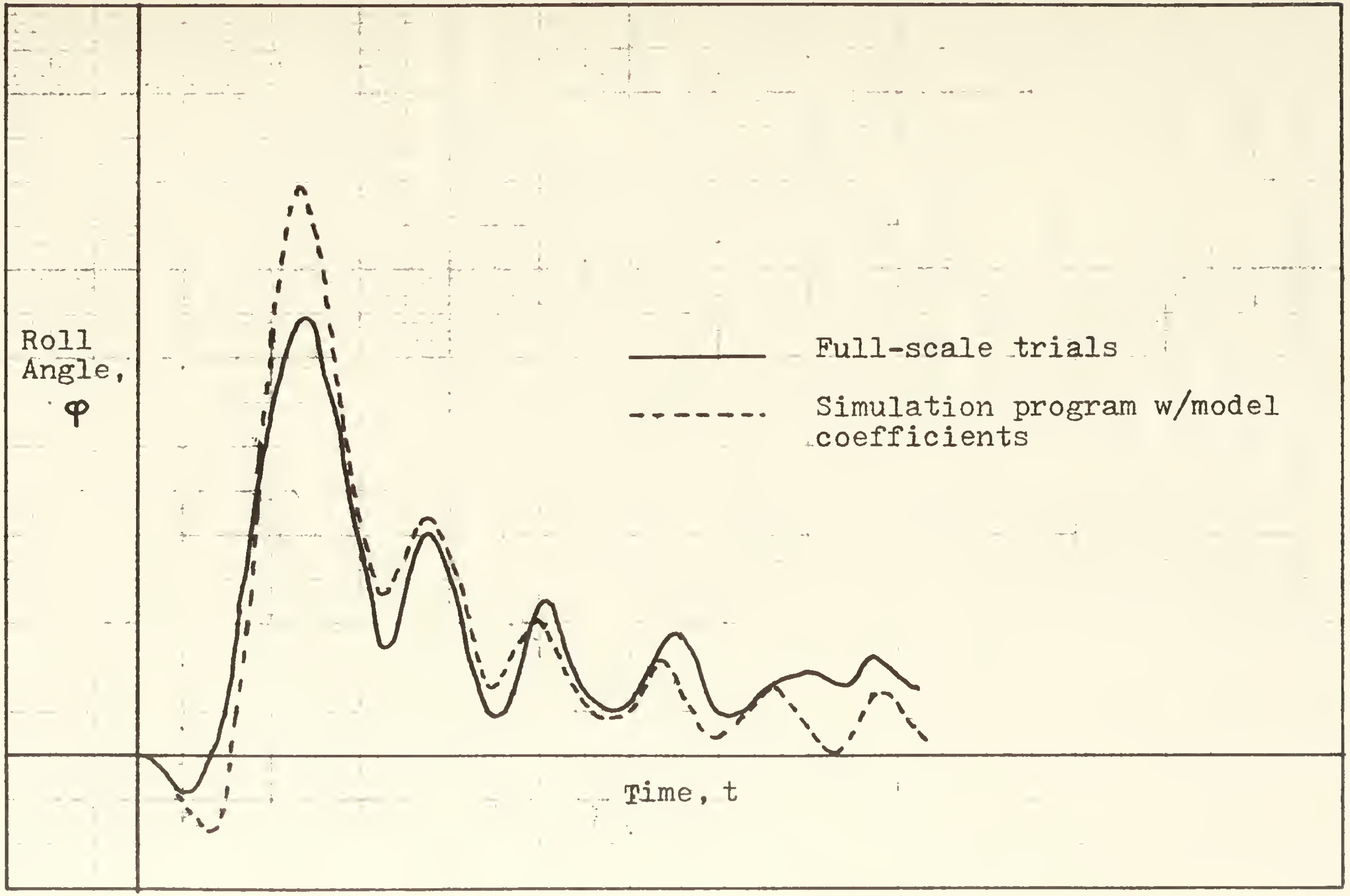

Figure 2-2

during the full scale trials could vary slightly from this value and, as will be seen in Chapter III, the maximum roll angle is very sensitive to small changes in GM. However, for other submarines and sequences, the predicted maximum roll angles were less than the full scale values and therefore, this somewhat indicates that the difficulty in predicting the snap roll angle does not lie entirely in the model or technique used. It will also be noted that the model results increasingly deviate from the full-scale trials as t increases. This can be attributed to two causes. First, the full-scale trials are performed with a spiraling maneuver. In order to save computer time and since we were only interested in the maximum roll angle, the computer simulation runs were only performed for 180° turns. Therefore, the rudder angle was taken off earlier for the model and the roll angle decreased more rapidly for large t. Secondly, at slower speeds (large t), the cross coupling effect of pitch and heave on roll is more pronounced than at higher speeds. Since we are using only four degree of freedom equations, the effect of pitch and heave at low speeds is lost.

CHAPTER III - METHODS FOR REDUCING SNAPROLL

III.1 GENERAL

There are a number of techniques and methods which have been proposed to reduce snap roll in submarines (1). Four of these alternatives will be investigated here. As was indicated in the introduction, the choice of these alternatives was based on the needs of the design community and time constraints. The four methods chosen can be categorized under one of two headings. The first concerns changes in the naval architectural characteristics of the design. Increasing GM and reducing sail size are classed under this category. The second category involves those alternatives which make use of an automatic ship control system. Rudder sequencing and speed reduction fall under this classification. Each method of snap roll reduction will be investigated individually in the following sections.

III.2 METACENTRIC HEIGHT

The metacentric height is directly related to the righting moment, K_φ, by the relation,

$$K_\varphi = -W \cdot GM$$

where W is the submerged displacement. This term is somewhat disguised in the governing equations in Appendix A. In the K-equation,

$$K_{\varphi} = z_B \cdot B$$

where B is the buoyancy, and equal to the submerged displacement at neutral buoyancy, and z_B is the metacentric height. It can be shown that, as a first approximation, the maximum roll angle is inversely proportional to the metacentric height (1). Therefore, it would be advantageous to increase the metacentric height in order to reduce the effect of snap roll. However, this could prove to be a difficult design task, since it would involve adding weight or rearranging weights in an already weight-limited design.

The metacentric height for several submarines was increased by 0.25, 0.50, 0.75, 1.0 and 1.5 feet. A non-dimensional plot of the results are shown in figure 3-1. It should be noted that the amount of rudder used and u_o were different for each vehicle. This indicates that the percent decrease in the maximum roll angle is essentially independent of u_o and the maximum ordered rudder angle for a given GM change. The curve is asymptotic for greater than 100% changes in metacentric height. Therefore, increases in the metacentric height of greater than about 100% will yield no greater than approximately an 82% decrease in the maximum roll angle.

As an example, a 30% increase in GM yields a 33% decrease in φ_{max}. For a typical submarine, this translates into a 10° decrease in the maximum roll angle for an approx-

The Effect of GM on Snap Roll

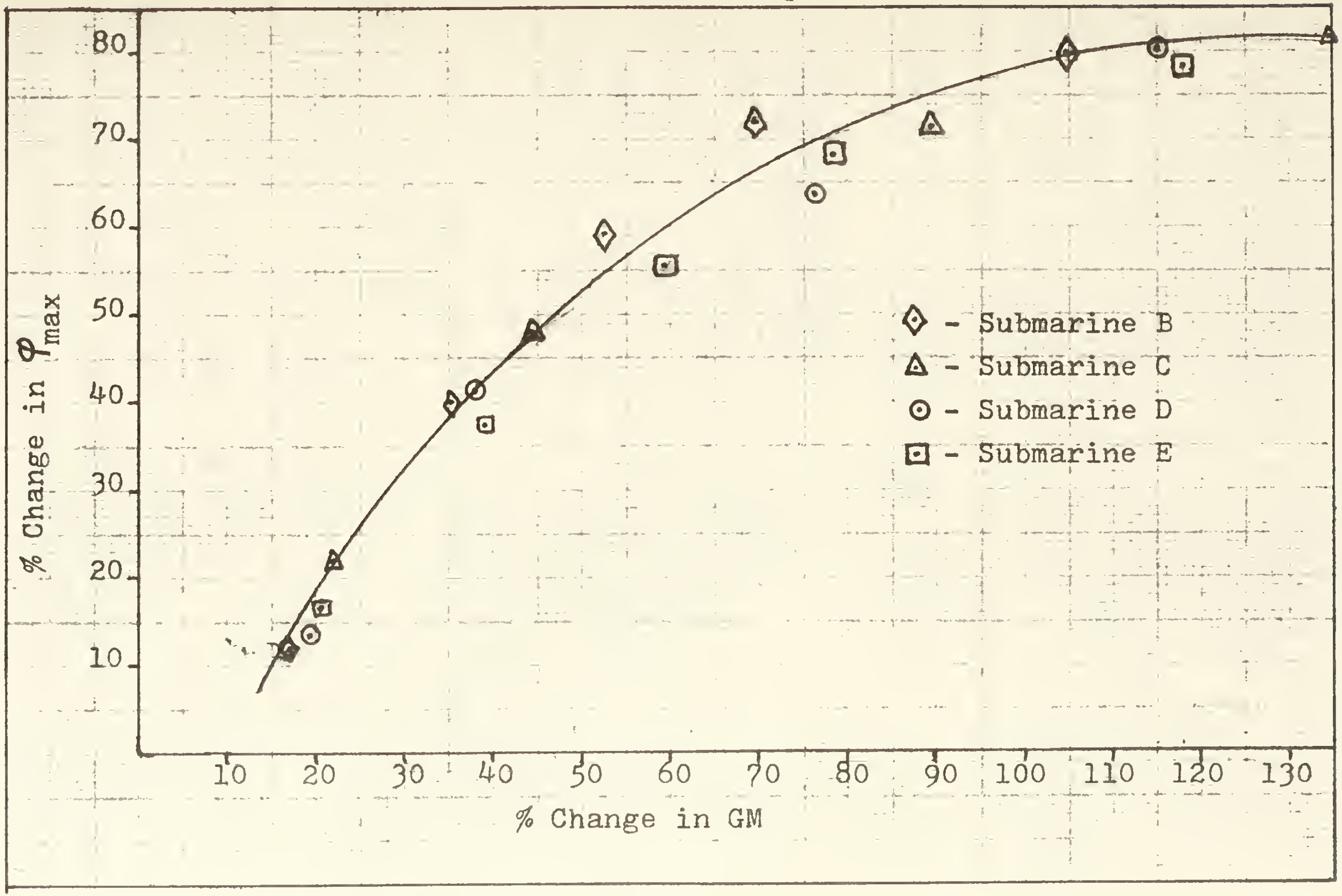

Figure 3-1

imately 4 inch increase in the metacentric height. It can be seen that large decreases in the snap roll angle can be realized for relatively small changes in GM. This is evident in figure 3-2. This is a plot for a typical submarine which shows the decrease in the maximum roll angle for increasing GM. An average 5.75° decrease in φ_{max} is gained for each 3 inch increase in GM. A point of minor interest shown in this plot is that the maximum roll angle occurs earlier in the turn as GM increases.

III.3 RUDDER SEQUENCING

Rudder sequencing means to defer the full rudder angle ordered, until some predetermined speed loss has been reached. It is expected that by limiting the initial rudder angle, the maximum roll angle will be reduced. This is based on the fact that the roll angle is directly proportional to the rudder deflection. Rudder sequencing is envisioned as part of an automatic ship control system; however, the method of rudder sequencing is simple enough to be employed as a manual process. One disadvantage of this method of snap roll reduction is that we might expect the transfer to increase. However, such an increase is normally no more than one half to one ship's length. The procedure employed in this thesis to investigate this roll reduction method involves reducing the maximum ordered rudder angle by 5, 10, 15 and 20 degrees.

Effect of Reducing GM on a Typical Submarine

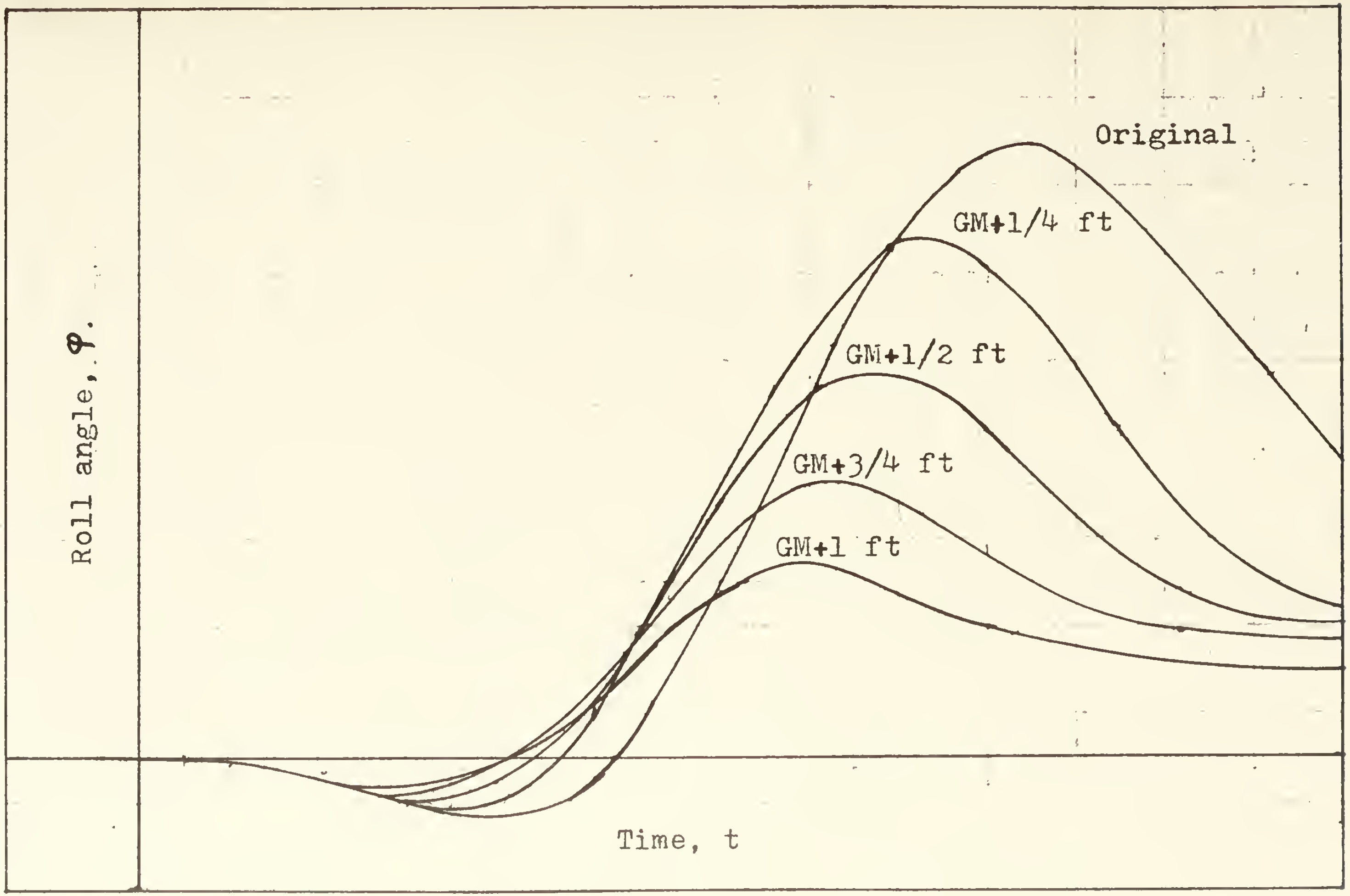

Figure 3-2

5

The predetermined speed loss criteria is to maintain the reduced rudder deflection until the speed in the turn has been reduced to <u>one third</u> that of the initial speed into the turn. This is depicted in figure 3-3. The selection of this speed criteria was based on the fact that the steady speed in a fully developed turn is approximately one third of the initial speed. The rudder deflection rate was held constant for all submarines and all sequences at 4 deg/sec.

The results are depicted in figure 3-4. As can be seen from the plot, this method of snap roll reduction is effective. However, relatively large rudder angle reductions are needed to produce small changes in φ_{max}. For example, a 10% reduction in the maximum roll angle requires a 23% reduction in the initial rudder angle. The use of rudder sequencing for a typical submarine is shown in figure 3-5. It can be seen that a greater decrease in φ_{max} <u>per degree of rudder</u> is gained for the 20° rudder sequence than for the 30° rudder sequence. It should be noted that some of the figure 3-4 data points were based on the same submarine at different initial speeds, i.e. 24 and 30 knots. Therefore, rudder sequencing seems to be independent of speed, in the sense that the same benefit is received for a particular rudder sequence despite the initial speed. For example, if a 20 degree rudder sequence reduces snap roll by 15% for u_o = 20 kts, then the same sequence will also reduce snap roll by 15% for a u_o = 30 kts.

Rudder Sequencing

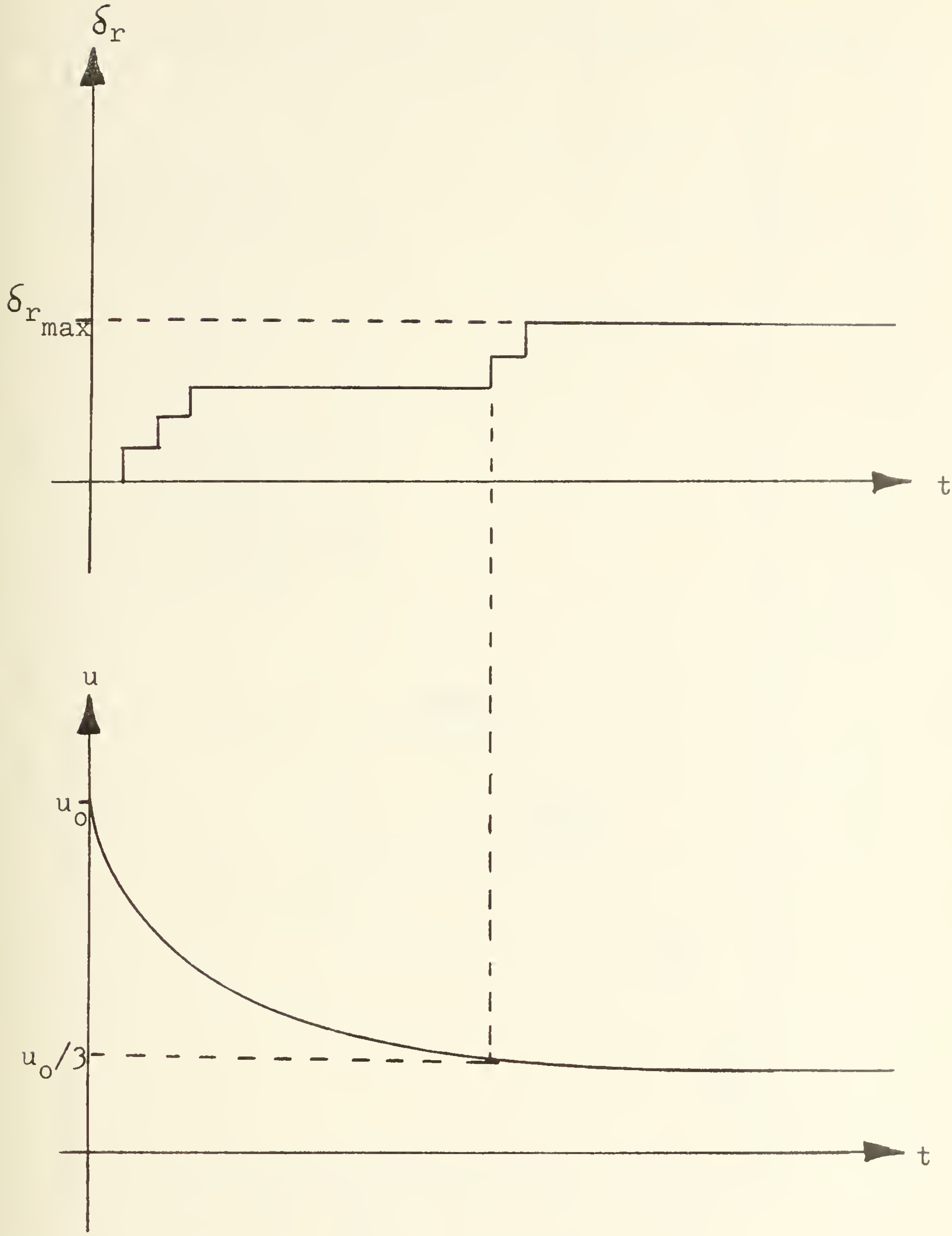

Figure 3-3

Effect of Rudder Sequencing on Snap Roll

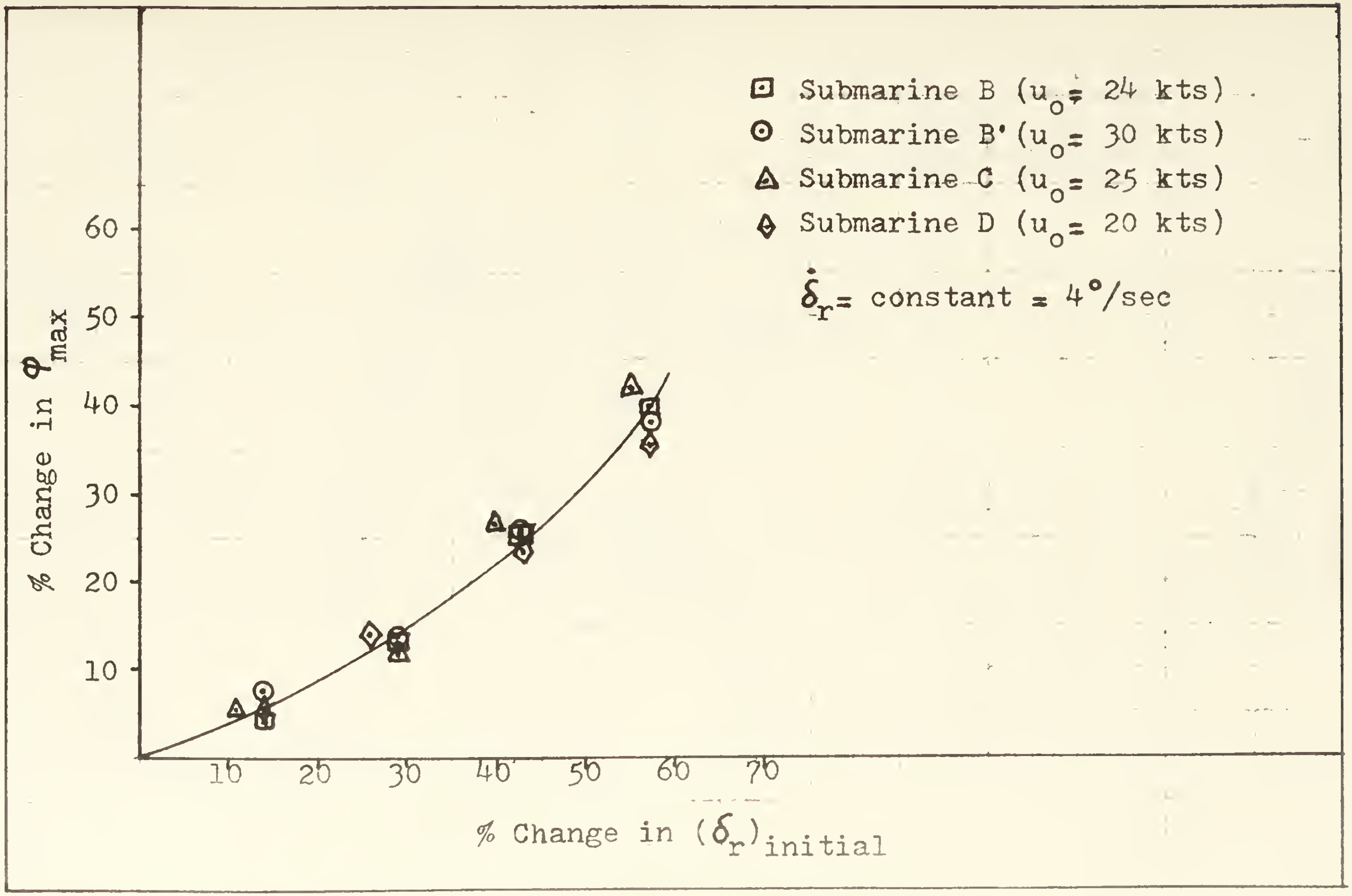

Figure 3-4

Rudder Sequence for a Typical Submarine

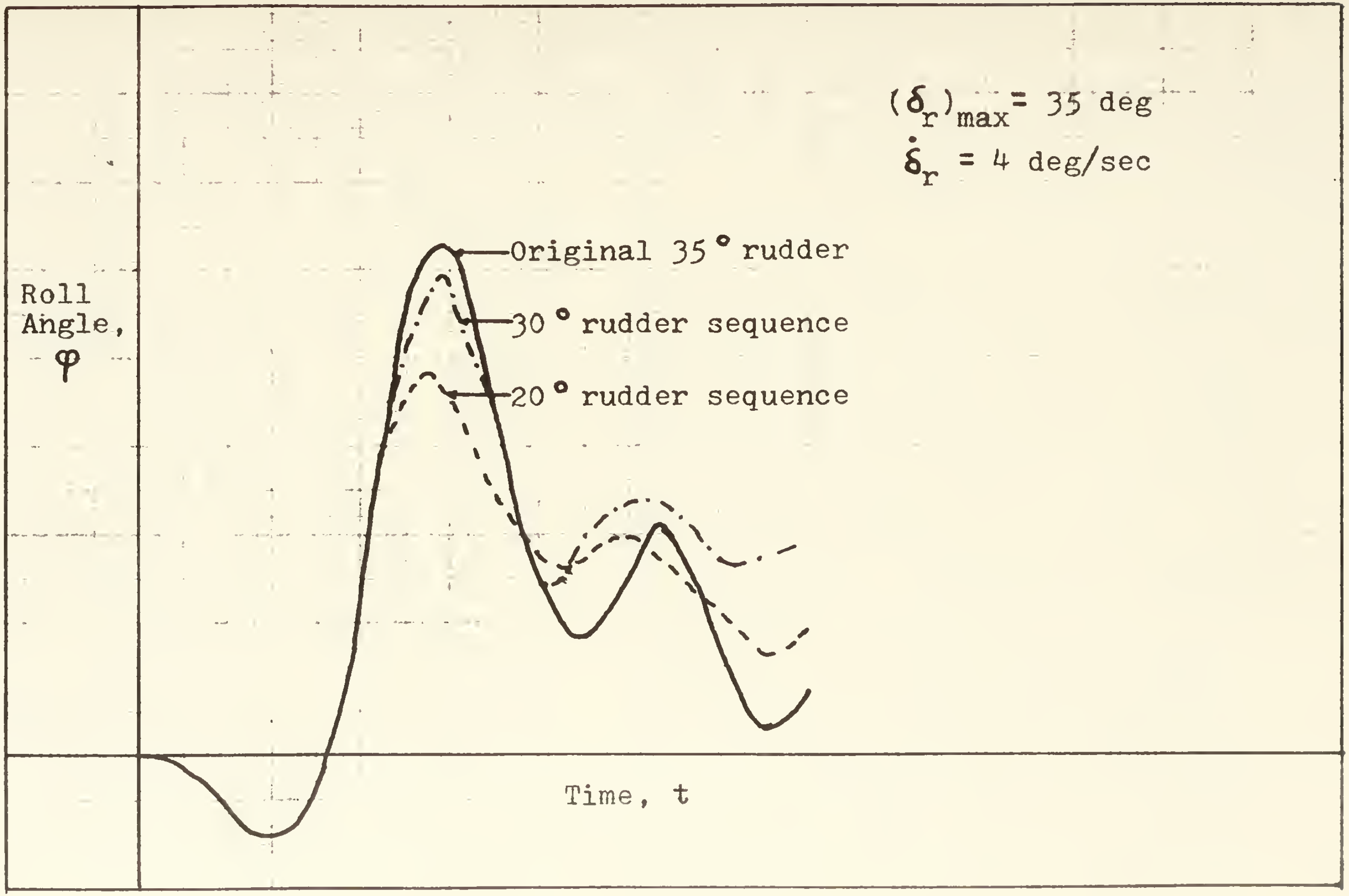

Figure 3-5

III.4 SAIL SIZE

A submarine sail produces a large roll moment which is caused by the sideslip velocity. The magnitude of the roll moment varies widely from submarine to submarine, primarily due to the extensive variation in sail location and size. The sail dimensions are primarily derived from considerations for the quantity of equipment and the dimensions of the equipment to be placed in the sail. With the exception of the fairwater, which encloses the sail, the naval architect pays little attention to the hydrodynamic considerations of sail design. It was felt that if the sail size could be reduced, that a corresponding reduction in snap roll would be realized.

An investigation was performed to determine if the chord or the span had the greater impact on snap roll. Figure 3-6 shows the results. Obviously, a change in the span will have a greater impact on roll, as evidenced by the larger slopes for the three coefficients considered. This result was anticipated since the moment arm associated with the spanwise location of the center of pressure will have a greater influence on roll than the moment arm associated with the chordwise location. Additionally, a reduction in span will produce a corresponding reduction in

Reduction in Roll Moment Due to Sail Size Reduction

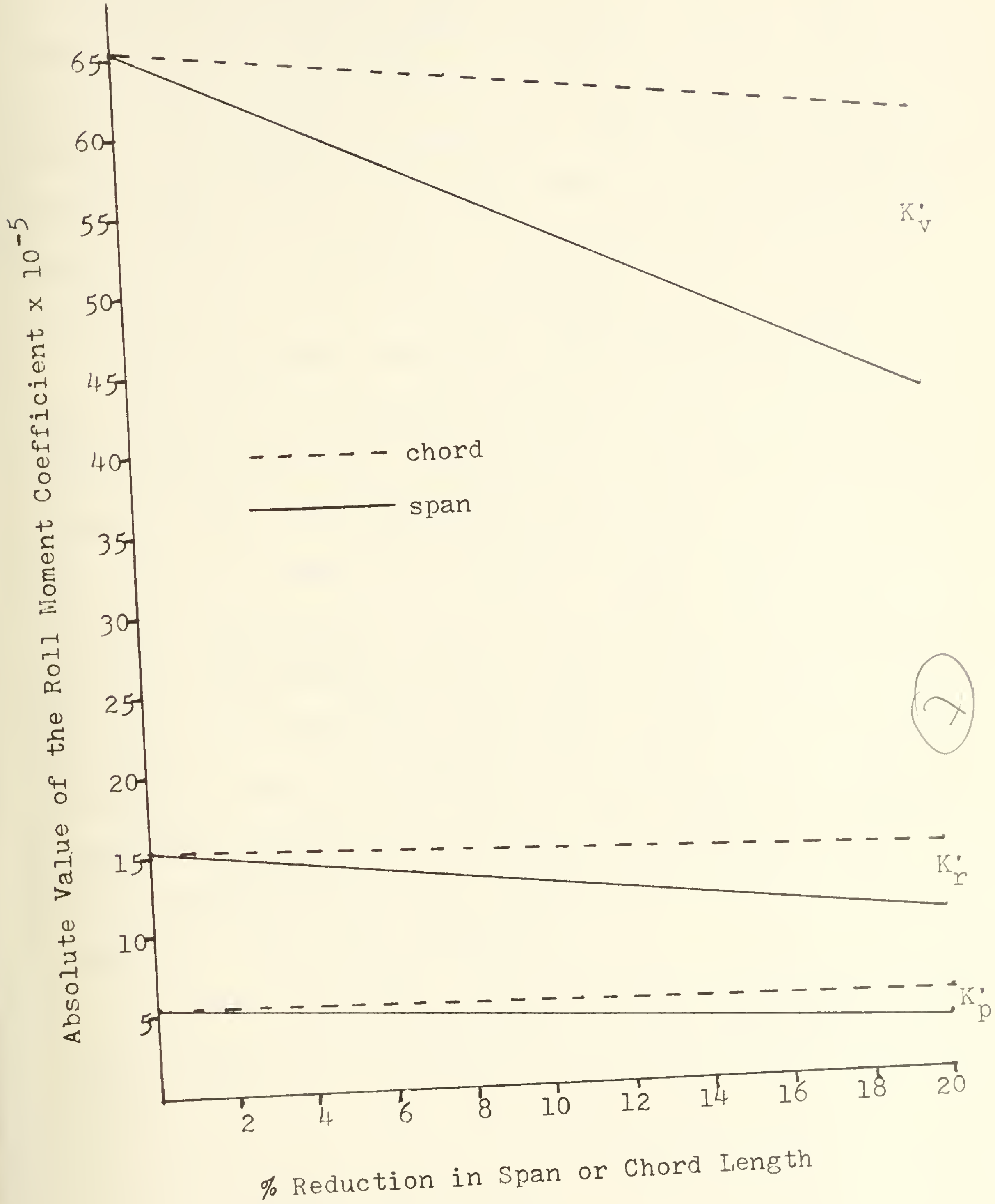

Figure 3-6

the lift due to the lower aspect ratio. A decrease in the chord would obviously increase the aspect ratio. Consequently, the span was reduced by 1, 5, 10, 15, 20 and 30 percent for a number of submarines. The results from these computer runs will be presented; however, first the method used to predict the impact on the hydrodynamic coefficients of reducing the span will be presented.

The approximate method for determining the impact of the sail on the hydrodynamic coefficients was partially derived from Abkowitz's presentation for a lifting foil in reference (4). This approximate method involves determining the Y force, N moment and K moment at the sail as a result of velocity disturbances v, r, and p. These coefficients are reduced correspondingly for a reduction in the sail size. The new sail coefficients are added to the original coefficients, less the original sail contribution. Then the equations of motion are solved for φ. Figure 3-7 defines the coordinate system, the variables involved in the derivation, and sign conventions.

For most purposes, the sail x and y velocity components are the same as the submarines's x and y velocity components, i.e.

$$u = u_s \qquad v = v_s.$$

The Coordinate System and Sign Conventions

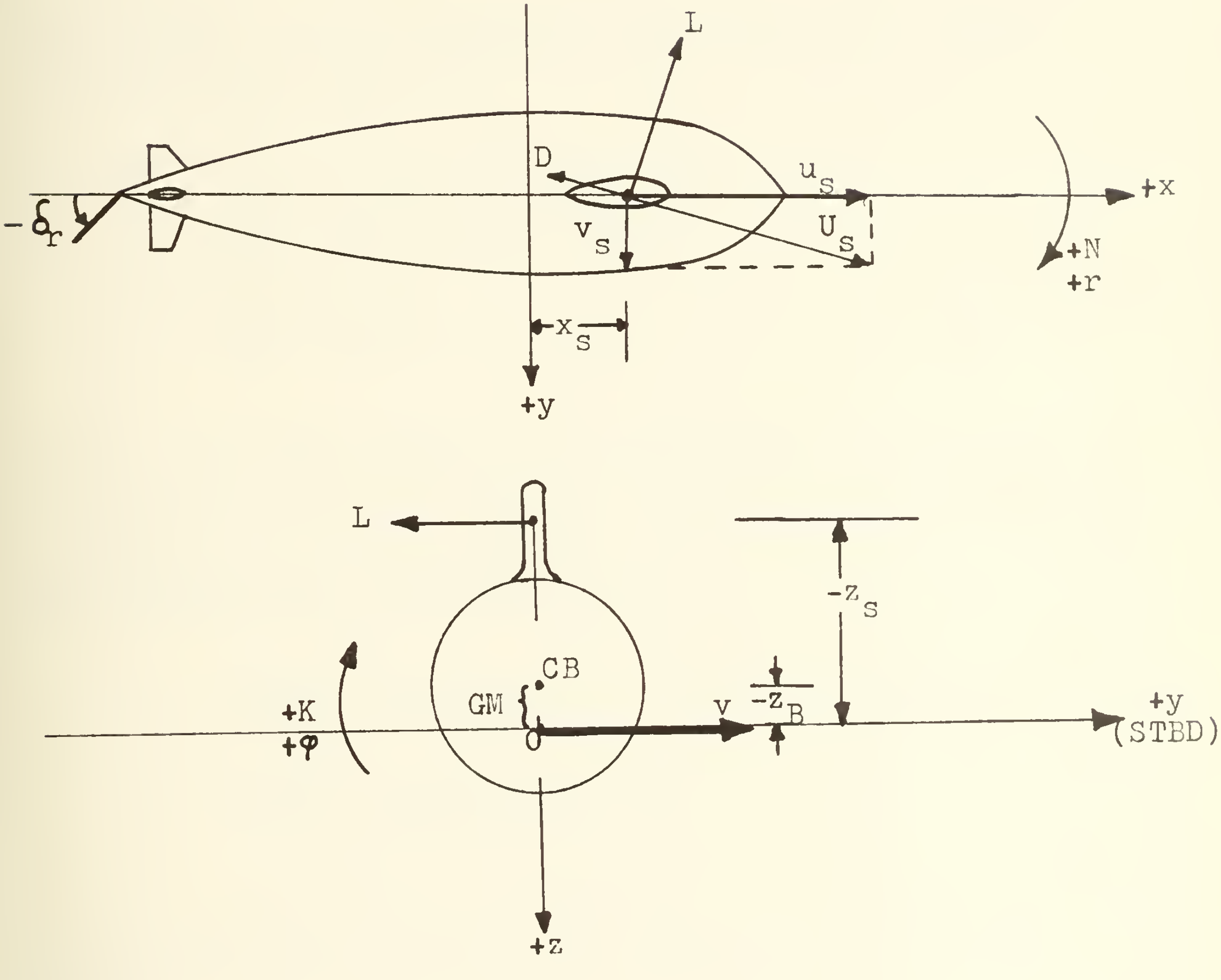

Figure 3-7

The subscript "s" denotes the local condition at the sail. The angle of attack produced at the sail by the transverse velocity is

$$\beta = \tan^{-1} \frac{v_s}{u_s} = \tan^{-1} \frac{v}{u} .$$

Consequently, we can calculate the Y force and N moment produced at the sail as

$$Y_s = - (L \cos\beta + D \sin\beta)$$

$$N_s = Y_s \cdot x_s$$

where L and D are the lift and drag forces on the sail, respectively. In terms of their respective non-dimensional coefficients, the lift and drag are

$$L = C_L \cdot \tfrac{1}{2} A_s U_s^2 = C_L \cdot \tfrac{1}{2}\rho A_s (u_s^2 + v_s^2)$$

$$D = C_D \cdot \tfrac{1}{2}\rho A_s (u_s^2 + v_s^2)$$

where A_s is the projected sail area and ρ is the fluid density.

From airfoil theory (6), the slope of the lift coefficient curve versus the angle of attack is shown to be approximated by

$$\frac{\partial C_L}{\partial \beta} = \frac{2\pi}{1 + 2/AR} \; .$$

AR is the "reflected aspect ratio." This is equal to twice the mean span divided by the mean chord, which corrects for the proper pressure distribution on the sail. Using the above relations, it can be shown that the Y force on the sail due to a lateral velocity, v, is

$$(Y_v)_s = -\tfrac{1}{2}\rho A_s u_s \left(\frac{2\pi}{1 + 2/AR} + C_D\right)$$

and the N moment due to v is

$$(N_v)_s = (Y_v)_s \cdot x_s \; .$$

Similarly, for the K moment

$$(K_v)_s = (Y_v)_s \cdot z_s$$

For the effect of an angular velocity, v, where $v_s = x_s \cdot r$, we find

$$Y_s \text{ due to } r = (Y_v)_s \cdot x_s \cdot r$$

$$N_s \text{ due to } r = (Y_v)_s \cdot x_s^2 \cdot r \qquad \text{and}$$

$$K_s \text{ due to } r = (Y_v)_s \cdot x_s \cdot z_s \cdot r \quad .$$

Taking the derivative of these coefficients with respect to r,

$$(Y_r)_s = (Y_v)_s \cdot x_s$$

$$(N_r)_s = (Y_v)_s \cdot x_s^{\,2}$$

$$(K_r)_s = (Y_v)_s \cdot x_s \cdot z_s \quad .$$

A similar process for the contribution of an angular velocity with $v_s = -z_s \cdot p$, yields

$$(Y_p)_s = -(Y_v)_s \cdot z_s$$

$$(N_p)_s = -(Y_v)_s \cdot x_s \cdot z_s$$

$$(K_p)_s = -(Y_v)_s \cdot z_s^{\,2} \quad .$$

A similar derivation provides the hydrodynamic coefficients for the sail with respect to the accelerations $\dot{v}$, $\dot{r}$ and $\dot{p}$. The key coefficient is, of course, $(Y_{\dot{v}})_s$. Based on the added mass calculation for a flat plate of dimensions s/2 and c, $(Y_{\dot{v}})_s$ is

$$(Y_{\dot{v}})_s = -\tfrac{1}{2}\left[\frac{\pi \rho s^2 \; c^2}{4\sqrt{s^2 + c^2}}\right] \quad .$$

This form of $(Y_{\dot{v}})_s$ is used with a correction factor developed by Abkowitz (1977) and finally becomes

$$(Y_{\dot{v}})_s = -\tfrac{1}{2}\left[\frac{\pi\rho\bar{s}^2\,\bar{c}^2}{4\sqrt{\bar{s}^2 + \bar{c}^2}}\right]\left[1 - \frac{0.54}{(1 + \bar{s}/\bar{c} + \bar{c}/\bar{s})}\right]$$

where $\bar{s}$ is the mean "reflected span" and $\bar{c}$ is the mean chord. The mean "reflected span" is directly related to the "reflected aspect ratio", in that, it corrects for the proper pressure distribution on the sail.

Since x_s and z_s are the distances from the center of pressure to the center of gravity along their respective axes, the following relations (7) proved beneficial,

$$(CP)_{\bar{c}} = 0.25 \cdot \bar{c}$$

$$(CP)_{\bar{s}} = (4/3)\pi \cdot \bar{s} \quad .$$

$(CP)_{\bar{c}}$ is the distance to the center of pressure chordwise location from the sail's leading edge. Similarly, $(CP)_{\bar{s}}$ is the distance from the root chord to the center of pressure.

Employing the above described procedure and equations, new coefficients were developed for reduction in sail size for a number of submarines. The results presented in figure 3-8 are for two different submarines, with what are

Change in the Snap Roll Angle due to a Reduction in Span

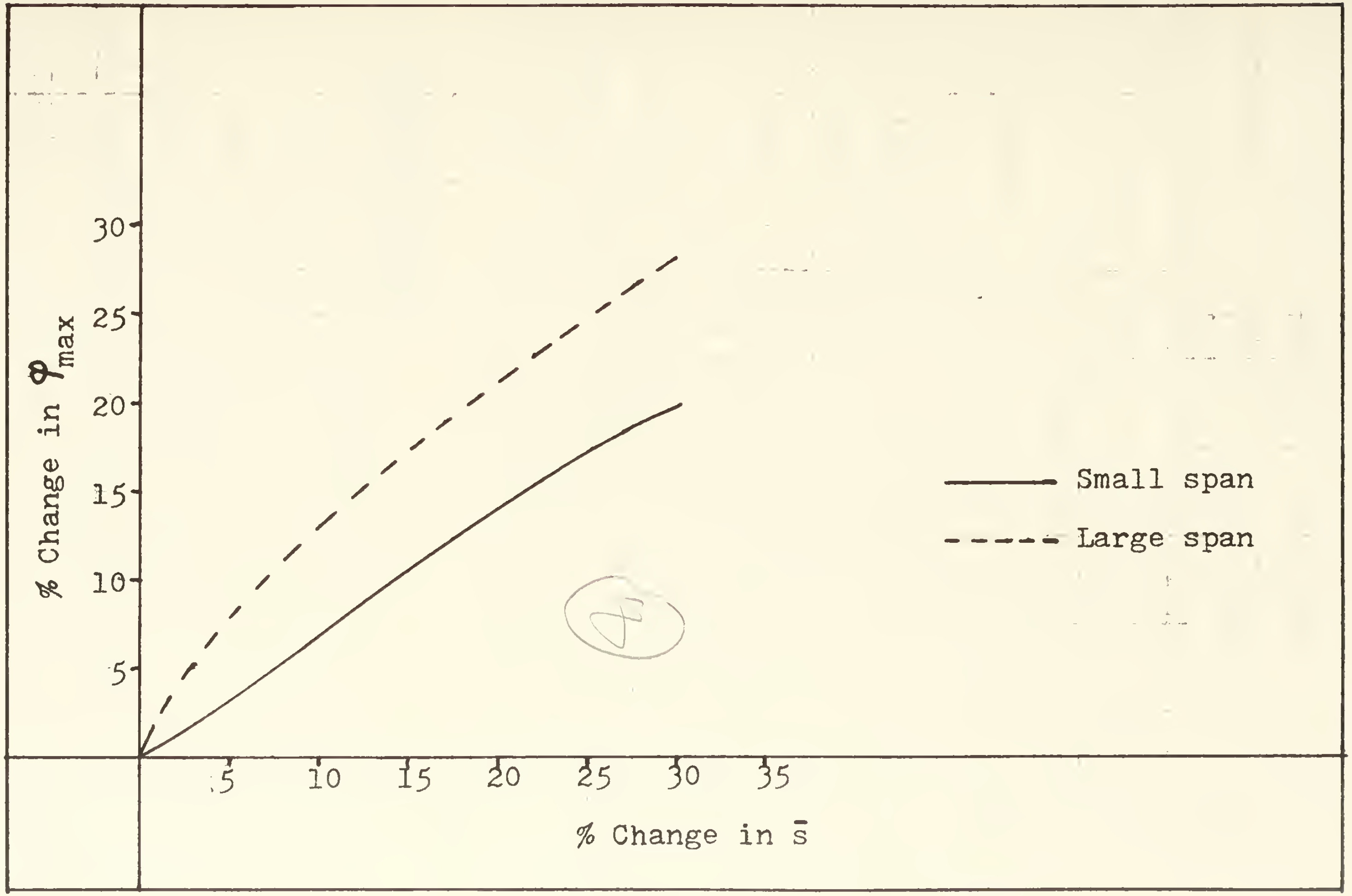

Figure 3-8

termed large and small spans. "Large" and "small" relate to the relative size of the sails when compared to similar submarine designs. It can be seen that a greater benefit is realized by the larger sail. The maximum reduction applied to the span was restricted to 30%. Any larger reduction in span would not be feasible due to physical restrictions, such as minimum periscope height requirements.

The decrease in the maximum roll angle through sail size reduction was not as large as was expected. It is believed that this may be explained by the fact that the sail contributes to the roll damping at the same time that it is acting as a roll producer. Therefore, by reducing the sail size, the roll damping is reduced, as well as the roll production. Fortunately, as indicated by figure 3-8, roll damping appears to decrease less rapidly than roll production for a given sail size reduction.

III.5 SPEED REDUCTION

It has been well established that the forward velocity, u, has a strong influence on the roll angle when in a turn. It can be shown that the maximum roll angle is proportional to the square of the velocity. Therefore, it is expected that small speed reductions will give significant reductions in φ_{max}. This is an important point, since if a submarine

is designed for a maximum speed in excess of 30 kts, then the operating community would like to be able to take full advantage of this speed in various maneuvers, as well as in a straight line motion. The greatest difficulty with snap roll occurs at a submarine's top speed. It would be attractive, from an operational point of view, to be able to enter a turn at almost maximum speed. With this in mind, several submarines were simulated to enter turns at from one to four knots below their maximum speed. The results are presented in figure 3-9. As was expected, small changes in the velocity produce relatively large reductions in snap roll. For example, a 10% reduction in u, decreases the maximum roll angle by almost 20%. This method of reducing the effect of snap roll is equally feasible for use in an automatic control system or as a manual method of operation.

Percent Change in the Snap Roll Angle for Small Reductions in u

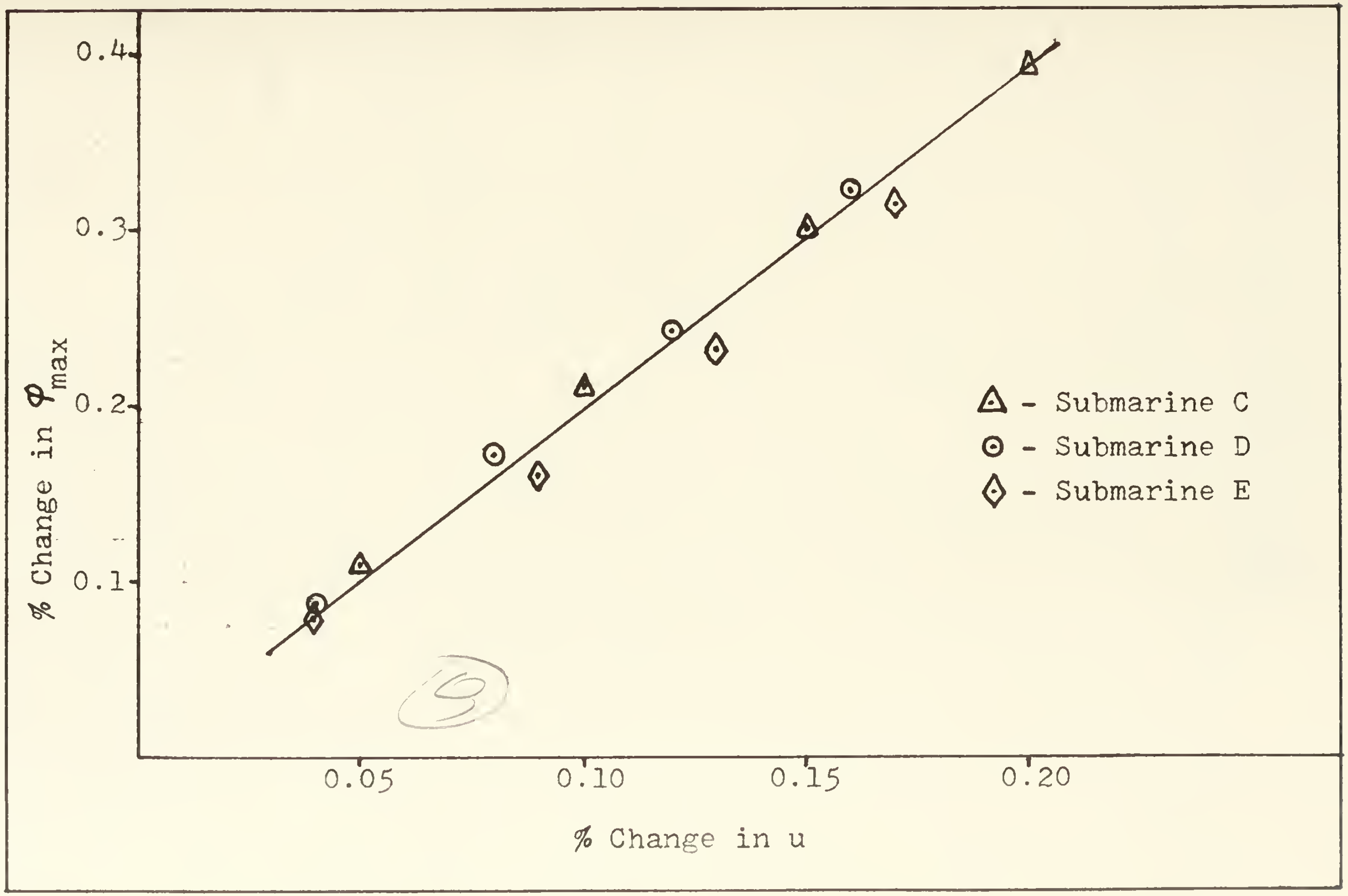

Figure 3-9

CHAPTER IV - CONCLUSIONS AND RECOMMENDATIONS

IV.1 CONCLUSIONS AND RECOMMENDATIONS

From this study, it must be concluded that we can, with sufficient engineering accuracy, predict full-scale submarine dynamic performance, at least in roll, from model data. This is accomplished through the use of a computer simulation model. This is of importance to the submarine designer for three reasons. First, this prediction technique provides the ship designer with an invaluable design tool. The stability characteristics of a multi-million dollar submarine design can be accurately predicted prior to making a major commitment to the procurement effort. The ship designer now can proceed with confidence on a design, at least from a dynamic stability standpoint. Secondly, the computer model can be used by the submarine designer to perturbate and iterate on a particular baseline design. Consequently, such a technique could be employed to optimize a design with respect to its stability criteria. Finally, this method of predicting stability performance can be employed by the designer to establish design criteria. Without a foreknowledge of a submarine's performance in a particular maneuver, the ship designer can test past similar designs to establish some pattern of performance and thereby derive a stability performance criteria.

It is apparent from the results of Chapter III that a small additional amount of GM in any submarine design will significantly reduce snap roll. Bearing this in mind, it would seem appropriate to suggest that the U.S. Navy revise its present design criteria for GM. The present criteria requires a minimum metacentric height, which does not work well for all designs from a stability standpoint. It is proposed that the revised design criteria for GM be based on a maximum acceptable snap roll angle. This would be a more flexible design criteria which could be tailored to individual designs. More importantly, it would couple a performance measure to the naval architectural characteristics. It would also accomodate for the variations in size of the roll moment producers which are encountered from design to design, i.e. the larger the roll moment producers, the larger the required GM to minimize the effect of snap roll.

The results of the other methods considered in Chapter III suggest obvious measures to be taken in an effort to reduce snap roll; these are to reduce speed, reduce initial rudder angle and reduce sail size. As was indicated in Section III.4, the reduction of sail size was not as effective in reducing snap roll as was anticipated. The possible reason for this was also suggested in Section III.4. However, a more thorough study in this area is certainly warranted. This is particularly true, in view of the fact

that an approximate method was used for predicting the appropriate hydrodynamic coefficients for the contribution of the sail to snap roll. A more precise method of prediction, coupled with a careful consideration of the total effect of the sail and sailplanes on roll stability, could lead to a better insight into the causes and minimization of snap roll. Other recommendations for future investigations are:

- The use of differential sailplanes and/or sternplanes as roll control fins.
- The use of unbalanced rudders.
- The effect of increasing the automation of submarine control systems on stability characteristics.

Three areas of improvement for the computer simulation model are proposed. The first is the extension of the present model to use the six degrees of freedom equations of motion. This should not only improve the accuracy of the results obtained from the present four degree of freedom program, but it would also allow for the investigation of more complex maneuvers and dynamic reactions, e.g. squat with trim plus roll. Secondly, the model should be modified to permit for more ease in its use by submarine designers. The less complex the input requirements and the more lucid

the output, the wider use the computer simulation model will experience within the design community. Lastly, it is recommended that the present program and future versions be optimized with respect to computer time. With the increasing expense and the time-sharing requirements of computer facilities, wider use of this type of computer simulation model will be realized by an efficient use of available computer time.

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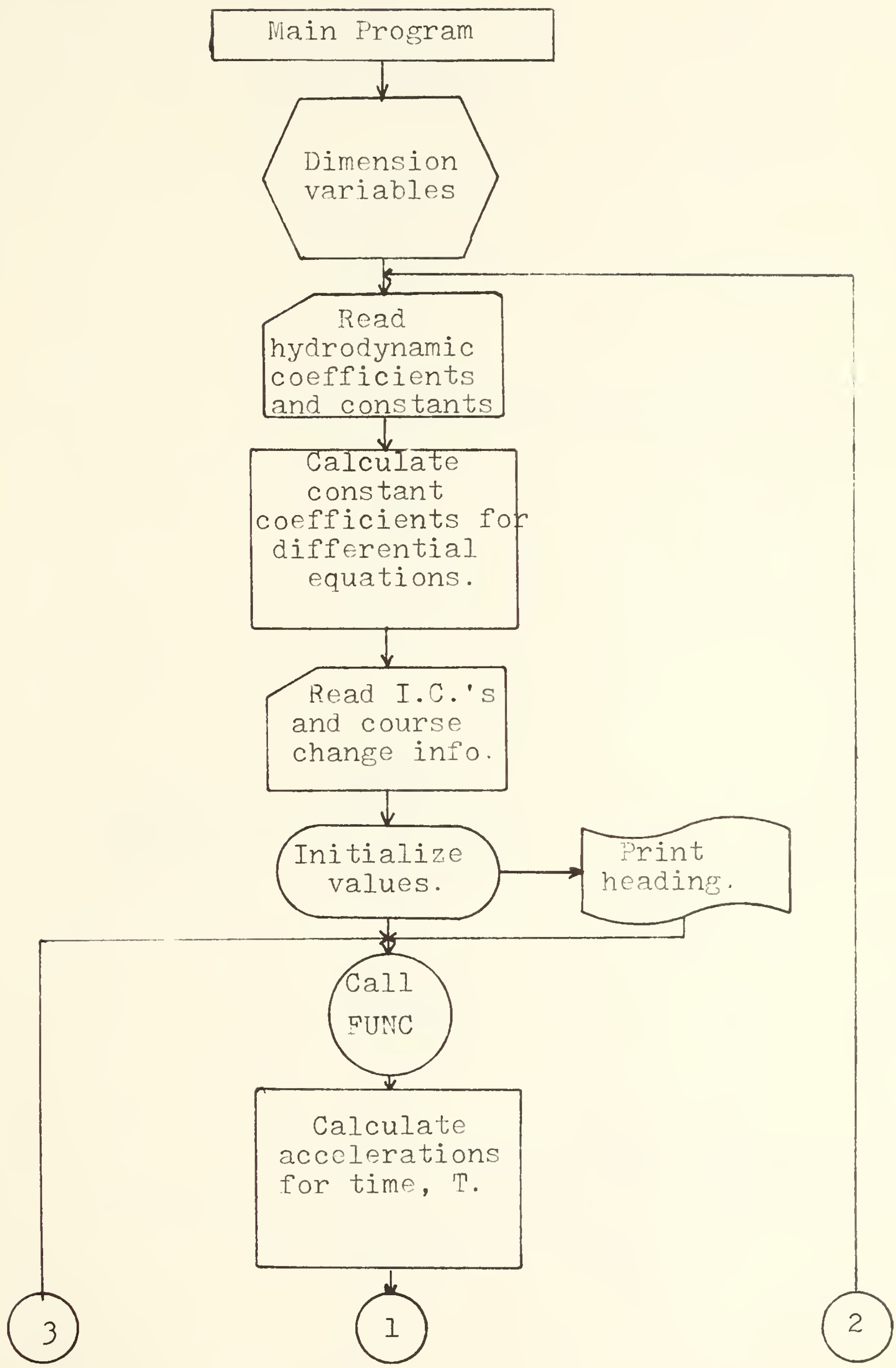
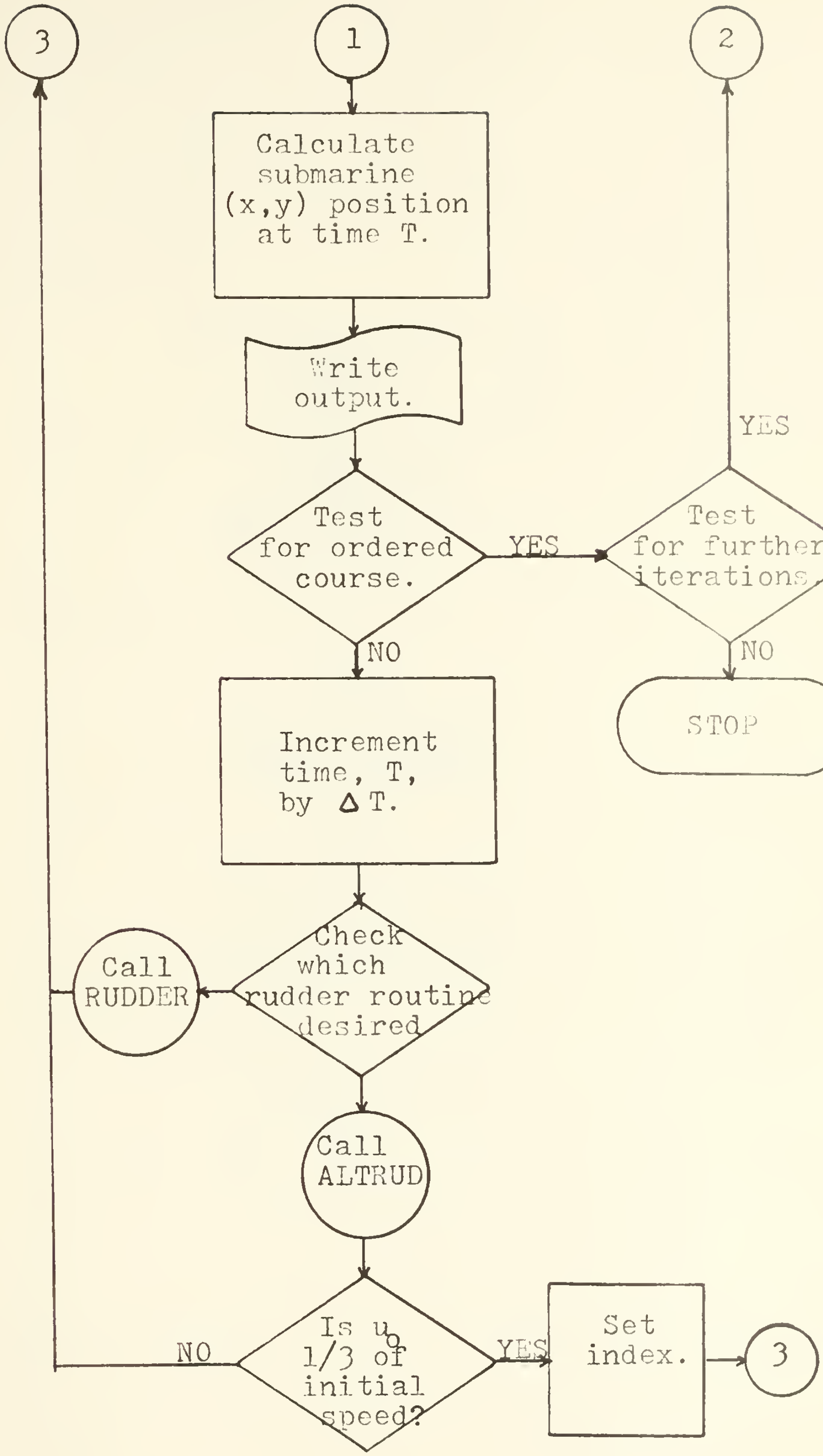
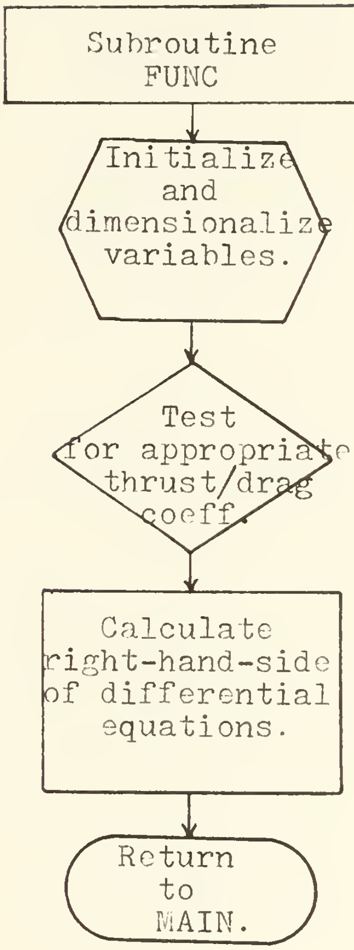
(8) ———, IBM System/360 and System/370 FORTRAN IV Language, GC28-6515-10, May 1974.

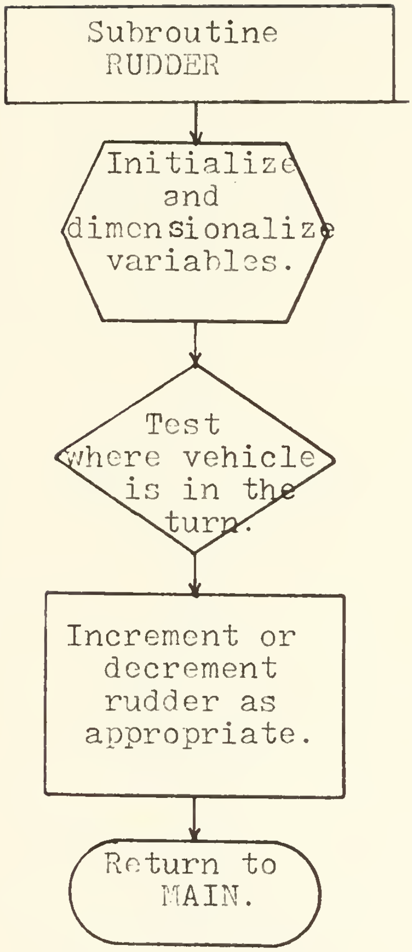
APPENDIX A

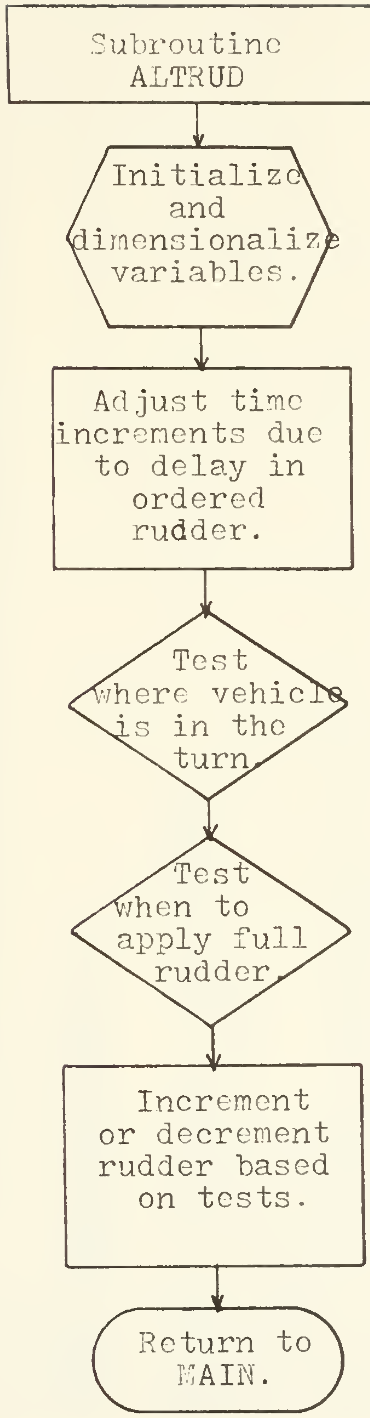
A.1 FLOW CHART




3
1
2
Calculate submarine (x,y) position at time T.
Write output.
Test for ordered course.
YES
Test for further iterations.
YES
NO
STOP
NO
Increment time, T, by Δ T.
Check which rudder routine desired
Call RUDDER
Call ALTRUD
Is u_o 1/3 of initial speed?
NO
YES
Set index.
3


Subroutine
FUNC
Initialize
and
dimensionalize
variables.
Test
for appropriate
thrust/drag
coeff.
Calculate
right-hand-side
of differential
equations.
Return
to
MAIN.


Subroutine RUDDER
Initialize and dimensionalize variables.
Test where vehicle is in the turn.
Increment or decrement rudder as appropriate.
Return to MAIN.


Subroutine ALTRUD
Initialize and dimensionalize variables.
Adjust time increments due to delay in ordered rudder.
Test where vehicle is in the turn.
Test when to apply full rudder.
Increment or decrement rudder based on tests.
Return to MAIN.

A.2 COMPUTER PROGRAM

```
C
C     MAIN PROGRAM --- READ DATA & INITIAL CONDITIONS & INITIAL VALUES.
C                      INITIALIZE & NON-DIMENSIONALIZE.  CALCULATE LINEAR D.E.
C                      COEFFICIENTS, ACCELERATIONS, VELOCITIES AND WRITE OUTPUT.
C
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(
     13),BI(3),CI(3),XCOEFF(16)
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ
      REAL ID(40)
      COMMON  T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG
      COMMON    XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG,
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,CELB,THETA,PH
     2I,A1,A2
      COMMON F1,F2,F3,F4,WT,B
      COMMON DELR,RHO,TR,RUD1,U3
      COMMON I1,I2
C
C     READ NON-DIMENSICNAL COEFFICIENTS
      INDEX=0
    1 READ(5,16,END=999) INDEX
   16 FORMAT (I2)
      IF(INDEX) 2,2,3
    3 READ(5,10,END=999)  (XCCEFF(I),I=1,16)
      READ(5,10)  (YCCEFF(I),I=1,22)
      READ(5,10)  (NCOEFF(I),I=1,22)
      READ(5,10)  (KCOEFF(I),I=1,17)
      READ(5,10)  (AI(I),I=1,3)
      READ(5,10)  (BI(I),I=1,3)
      READ(5,10)  (CI(I),I=1,3)
   10 FORMAT(D10.4)
      READ(5,11) A1,A2
   11 FORMAT(2D10.4)
C     READ CONSTANTS -- MASS, LENGTH, ETC.
      READ(5,12)  XG,YG,ZG
      READ(5,12)  XB,YB,ZB
```

```
      READ(5,12)  M,L,RHO                                              MAIN0037
      READ(5,12)  IX,IY,IZ                                             MAIN0038
      READ(5,12)  IXY,IXZ,IYZ                                          MAIN0039
   12 FORMAT(3D10.4)                                                   MAIN0040
      READ(5,13)  WT,B                                                 MAIN0041
   13 FCRMAT(2D10.4)                                                   MAIN0042
C     CALCULATE COEFFICIENT VALUES FOR THE FOUR LINEAR D.E.'S          MAIN0043
      E(1)=0.0                                                         MAIN0044
      D(1)=0.0                                                         MAIN0045
      A(2)=C.0                                                         MAIN0046
      A(4)=0.0                                                         MAIN0047
      A(1)=M-XCCEFF(4)                                                 MAIN0048
      C(1)=YG                                                          MAIN0049
      E(2)=M-YCOEFF(6)                                                 MAIN0050
      D(2)=(M*ZG)+(YCOEFF(2)*L)                                        MAIN0051
      C(2)=(M*XG)-(YCOEFF(1)*L)                                        MAIN0052
      C(3)=IZ-NCOEFF(1)                                                MAIN0053
      D(3)=IXZ+NCOEFF(2)                                               MAIN0054
      E(3)=(M*(XG/L**2))-NCOEFF(6)*(1/L)                               MAIN0055
      A(3)=M*(YG/L**2)                                                 MAIN0056
      E(4)=(M*(ZG/L**2))+KCOEFF(8)*(1/L)                               MAIN0057
      C(4)=IXZ-KCOEFF(2)                                               MAIN0058
      D(4)=IX-KCCEFF(1)                                                MAIN0059
      DENOM=A(1)*(E(2)*C(3)*D(4)+C(2)*D(3)*E(4)+D(2)*E(3)*C(4)-E(4)*C(3) MAIN0060
     1*D(2)-C(4)*D(3)*E(2)-D(4)*E(3)*C(2))+C(1)*(D(2)*A(3)*E(4)-D(4)*A(3 MAIN0061
     2)*E(2))                                                          MAIN0062
C     READ INITIAL CONDITIONS                                          MAIN0063
    2 READ(5,14,END=999)   (ID(I),I=1,40)                              MAIN0064
   14 FORMAT(40A1)                                                     MAIN0065
      READ(5,15)  U,V,W,UDT,VDT,WDT                                    MAIN0066
      READ(5,15)  P,Q,R,PDT,QDT,RDT                                    MAIN0067
      READ(5,12)  THETA,PHI,PSI                                        MAIN0068
      READ(5,12)  DELS,DELB,DELGM                                      MAIN0069
   15 FORMAT(6D10.4)                                                   MAIN0070
      ZB=ZB-DELGM                                                      MAIN0071
      UO=U                                                             MAIN0072
```

```
      U=U*1.6889
      V=V*1.6889
      W=W*1.6889
      UDT=UDT*1.6889
      VDT=VDT*1.6889
      WDT=WDT*1.6889
C     READ COURSE CHANGE INFORMATICN
      READ(5,15) TLAG,COURSE,RRATE,RUDAMT,RUD1,DELT
C     WRITE COURSE CHANGE INPUTS ANC HEADINGS
      WRITE(6,49)    (ID(I),I=1,40)
   49 FORMAT(1H1,39X,'**************************************************
     1*',/,40X,'*',49X,'*',/,40X,'*    ',40A1,'     *',/,40X,'*',49X,'*'
     2,/,40X,'**************************************************',///)
      WRITE(6,42)  UO,RRATE,COURSE,RUDAMT,DELT,RUD1
   42 FORMAT(33X,'UO =',D9.3,5X,'RUDDER RATE =',D9.3,5X,'COURSE CHANGE =
     1',D9.3,//,33X,'AMOUNT OF RUDDER =',D9.3,5X,'DELTA T =',D9.3,//,33X
     2,'RUD1 =',D9.3,/////)
      WRITE(6,41)
   41 FORMAT(5X,'T',9X,'U',9X,'V',9X,'R',8X,'PHI',7X,'UDT',7X,'VDT',7X,'
     1RDT',7X,'PDT',7X,'DELR',6X,'CRS',8X,'X',9X,'Y',//)
      WRITE(6,44)
   44 FORMAT(3X,'(SEC)',5X,'(KTS)',5X,'(KTS)',3X,'(DEG/SEC)',3X,'(DEG)',
     13X,'(KTS/HRS)',1X,'(KTS/HRS)',1X,'(DEG/SEC/',1X,'(DEG/SEC/',3X,'(D
     2EG)',5X,'(DEG)',5X,'(YDS)',5X,'(YDS)',/,72X,'SEC)',6X,'SEC)',////)
C     INITIALIZE TIME RUDDER QUANTITIES
      T=0.0D1
      X=0.0D1
      Y=0.0D1
      I1=0
      I2=0
      TR=0.0D+1
      UO=UO*1.6889
C     CONVERT ANGLES IN DEGREES TO RADIANS FOR INTERNAL USE.
      PHI=PHI/57.2958
      THETA=THETA/57.2958
      PSI=PSI/57.2958
```

```
      COURSE=COURSE/57.2958                                              MAIN0109
      RRATE=RRATE/57.2958                                                MAIN0110
      RUDAMT=RUDAMT/57.2958                                              MAIN0111
      RUD1=RUD1/57.2958                                                  MAIN0112
      RUD=DABS(RUDAMT)                                                   MAIN0113
      T2=(RUD/RRATE)+TLAG                                                MAIN0114
      U3=U0/0.3D+1                                                       MAIN0115
      CRS1=DABS(COURSE)                                                  MAIN0116
      RTIME=CRS1/RRATE                                                   MAIN0117
      DELR=0.0D1                                                         MAIN0118
      IF(RTIME.LE.TLAG) GO TO 20                                         MAIN0119
      IF(T.EQ.0.0D1) GO TO 31                                            MAIN0120
C     CALL "FUNC" SUBROUTINE AND CALCULATE ACCELERATIONS AND VELOCITIES  MAIN0121
C     AT TIME T + DELTA T.                                               MAIN0122
   30 PHI=PHI+(DELT*P)+((DELT**2)/2)*PDT                                 MAIN0123
      PSI=PSI+(DELT*R)+((DELT**2)/2)*RDT                                 MAIN0124
      U=U+DELT*UDT                                                       MAIN0125
      V=V+DELT*VDT                                                       MAIN0126
      R=R+DELT*RDT                                                       MAIN0127
      P=P+DELT*PDT                                                       MAIN0128
      CALL FUNC                                                          MAIN0129
C                                                                        MAIN0130
      UDT=(F1*(E(2)*C(3)*D(4)+C(2)*D(3)*E(4)+D(2)*E(3)*C(4)-E(4)*C(3)*D( MAIN0131
     12)-C(4)*D(3)*E(2)-D(4)*E(3)*C(2))+C(1)*(F2*E(3)*D(4)+E(2)*D(3)*F4+ MAIN0132
     2D(2)*F3*E(4)-F4*E(3)*D(2)-E(4)*D(3)*F2-D(4)*F3*E(2)))/DENOM        MAIN0133
C                                                                        MAIN0134
      VDT=(A(1)*(F2*C(3)*D(4)+C(2)*D(3)*F4+D(2)*F3*C(4)-F4*C(3)*D(2)-C(4 MAIN0135
     1)*D(3)*F2-D(4)*F3*C(2))-F1*(D(2)*A(3)*C(4)-D(4)*A(3)*C(2))+C(1)*(D MAIN0136
     2(2)*A(3)*F4-D(4)*A(3)*F2))/DENOM                                   MAIN0137
C                                                                        MAIN0138
      RDT=(A(1)*(E(2)*F3*D(4)+F2*D(3)*E(4)+D(2)*E(3)*F4-E(4)*F3*D(2)-F4* MAIN0139
     1D(3)*E(2)-D(4)*E(3)*F2)+F1*(D(2)*A(3)*E(4)-D(4)*A(3)*E(2)))/DENOM  MAIN0140
C                                                                        MAIN0141
      PDT=(A(1)*(E(2)*C(3)*F4+C(2)*F3*E(4)+F2*E(3)*C(4)-E(4)*C(3)*F2-C(4 MAIN0142
     1)*F3*E(2)-F4*E(3)*C(2))+C(1)*(F2*E(4)*A(3)-F4*A(3)*E(2))-F1*(C(2)* MAIN0143
     2A(3)*E(4)-C(4)*A(3)*E(2)))/DENOM                                   MAIN0144
```

```
C
C     CALCULATE POSITION OF SUBMARINE (X,Y,)      AT TIME T.  ORIGIN TAKEN
C     AT CG AT TIME = 0 .
      X=X+((V*DSIN(PSI)+U*DCOS(PSI))*DELT)
      Y=Y+((U*DSIN(PSI)-V*DCOS(PSI))*DELT)
C     WRITE OUTPUTS FOR TIME - T,
C     CONVERT ANGLES IN RADIANS TO DEGREES FOR OUTPUT.
   31 PDTO=PDT*57.2958
      RDTO=RDT*57.2958
      DELRO=DELR*57.2958
      PHIO=PHI*57.2958
      UOO=U/1.6889
      VO=V/1.6889
      PSIO=PSI*57.2958
      UDTO=UDT/1.6889
      VDTO=VDT/1.6889
      RO=R*57.2958
      XX=X/3.0
      YY=Y/3.0
      WRITE(6,43) T,UOO,VO,RO,PHIO,UDTO,VDTO,RDTO,PDTO,DELRO,PSIO,XX,YY
   43 FORMAT(13(1X,D9.3),/)
C
C     END CALCULATIONS WHEN ORDERED COURSE IS REACHED.
      CRS2=CRS1-DABS(PSI)
      IF(CRS2.LE.0.00435) GO TO 1
C
C     TIME INCREMENT ALGORITHM.
C
      IF(T.GE.TLAG) GO TO 21
      T=TLAG+DELT
      GO TO 22
   21 T=T+DELT
C     UPDATE RUDDER ANGLE FOR TIME T.
   22 IF(U.LE.U3) GO TO 23
      IF(RUD1.EQ.0.0D+1) GO TO 23
      CALL ALTRUD
```

```
      GO TO 30
   23 CALL RUDDER
      GC TO 30
C     ERROR MESSAGE.
   20 WRITE(6,40)
   40 FORMAT(///,30X,'***   ERROR   ***',//,10X,'THE TIME LAG SPECIFIED
     1IS GREATER OR EQUAL TO THE TIME ALLOWED FOR THE TURN.  CHECK TLAG,
     2',/,10X,' RRATE AND COURSE.')
  999 STOP
      END
```

```
C
      SUBROUTINE FUNC
C
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(
     13),BI(3),CI(3),XCOEFF(16)
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ
      COMMON  T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG
      COMMON    XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG,
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,DELB,THETA,PH
     2I,A1,A2
      COMMON F1,F2,F3,F4,WT,B
      COMMON DELR,RHC,TR,RUD1,U3
      COMMON I1,I2
C
C     SUBROUTINE FUNC CALCULATES THE THE RIGHT HAND SIDE OF THE FOUR NON-LINEAR
C     DIFFERENTIAL EQUATIONS.
C
      ETA=0.0D1
C     CALCULATE MAGNITUDE OF VECTOR VELOCITY.
      BIGU=DSQRT(U**2+V**2+W**2)
      ETA=UO/BIGU
      THETAR=THETA
      PHIR=PHI
      VW=DSQRT(V**2+W**2)
C     TEST FOR APPROPRIATE THRUST/DRAG COEFFICIENT
      IF(ETA.GE.A1) GO TO 10
      IF(ETA.GE.A2) GO TO 11
      AII=AI(3)
      BII=BI(3)
      CII=CI(3)
      GO TO 40
   11 AII=AI(2)
      BII=BI(2)
      CII=CI(2)
      GO TO 40
```

```
   10 AII=AI(1)                                                           FUNC0037
      BII=BI(1)                                                           FUNC0038
      CII=CI(1)                                                           FUNC0039
C      RIGHT-HAND-SIDE OF LINEAR D.E.'S FOR UDT,VDT,RDT AND PDT.          FUNC0040
C                                                                         FUNC0041
   40 F1=(L*(XCOEFF(1)*(Q**2)+XCOEFF(2)*(R**2)+XCOEFF(3)*R*P))+(XCOEFF(5 FUNC0042
     1)*V*R+XCOEFF(6)*W*Q)+((1/L)*(XCOEFF(7)*(U**2)+XCOEFF(8)*V**2+XCOEF FUNC0043
     2F(9)*W**2+AII*U**2+BII*U*UO+CII*UO**2))+((1/L)*(U**2)*(XCOEFF(10)* FUNC0044
     3(DELR**2)+XCOEFF(11)*(DELS**2)+XCOEFF(12)*(DELB**2)))+((1/L)*(ETA- FUNC0045
     41)*(XCOEFF(13)*(V**2)+XCOEFF(14)*(W**2)+XCOEFF(15)*(DELR*U)**2+XCO FUNC0046
     5EFF(16)*(DELS*U)**2))-((2/(RHO*(L**3)))*(WT-B)*DSIN(THETAR))-(M*(W FUNC0047
     6*Q-V*R-XG*(Q**2+R**2)+YG*P*Q+ZG*(P*R+Q)))                          FUNC0048
C                                                                         FUNC0049
      Z=V                                                                 FUNC0050
      IF(Z.EQ.0.0D1) Z=0.1D-15                                            FUNC0051
      F2=(L*(YCOEFF(3)*P*DABS(P)+YCOEFF(4)*P*Q+YCOEFF(5)*Q*R))+(YCOEFF(7 FUNC0052
     1)*Q*V+YCOEFF(8)*W*P+YCOEFF(9)*W*R+YCOEFF(10)*U*R+YCOEFF(11)*U*P+YC FUNC0053
     2OEFF(12)*U*DABS(R)*DELR+YCOEFF(13)*(V/DABS(Z))*DABS(VW)*DABS(R)+YC FUNC0054
     3OEFF(19)*U*P*(ETA-1))+((1/L)*(YCOEFF(14)*(U**2)+YCOEFF(15)*U*V+YCO FUNC0055
     4EFF(16)*V*DABS(VW)+YCOEFF(17)*V*W+YCOEFF(18)*(U**2)*DELR))+((2/(RH FUNC0056
     5O*L**3))*(WT-B)*DCOS(THETAR)*DCOS(PHIR))+(((1/L)*(ETA-1))*(YCOEFF( FUNC0057
     620)*U*V+YCOEFF(21)*V*DABS(VW)+YCOEFF(22)*DELR*(U**2)))-(M*(U*R-W*P FUNC0058
     7-YG*(R**2+P**2)+ZG*Q*R+XG*Q*P))                                    FUNC0059
C                                                                         FUNC0060
      F3=(NCOEFF(3)*P*Q+NCOEFF(4)*Q*R+NCOEFF(5)*R*DABS(R)+((1/L)*(NCOEFF FUNC0061
     1(7)*W*R+NCOEFF(8)*W*P+NCOEFF(9)*V*Q+NCOEFF(10)*U*P+NCOEFF(11)*U*R+ FUNC0062
     2NCOEFF(12)*U*DABS(R)*DELR+NCOEFF(13)*DABS(VW)*R))+((1/L**2)*(NCOEF FUNC0063
     3F(14)*U**2+NCOEFF(15)*U*V+NCOEFF(16)*V*DABS(VW)+NCOEFF(17)*W*V+NCO FUNC0064
     4EFF(18)*(U**2)*DELR))+((2/RHO*L**5))*((XG*WT-XB*B)*DCOS(THETAR)*DS FUNC0065
     5IN(PHIR)+(YG*WT-YB*B)*DSIN(THETAR)))+((1/L)*(ETA-1)*NCOEFF(19)*U*R FUNC0066
     6)+((1/L**2)*(ETA-1)*(NCOEFF(20)*U*V+NCOEFF(21)*V*DABS(VW)+NCOEFF(2 FUNC0067
     72)*DELR*U**2))-((IX-IY)*P*Q)+(IYZ*(P*R+QDT))-(IXY*(Q**2-P**2))-(IX FUNC0068
     8Z*P*R)-(M*(1/L**2)*(XG*(U*R-W*P)-YG*(W*Q-V*R)))                    FUNC0069
C                                                                         FUNC0070
      F4=(KCOEFF(3)*Q*R+KCOEFF(4)*P*Q+KCOEFF(5)*P*DABS(P))+((1/L)*(KCOEF FUNC0071
     1F(6)*U*P+KCOEFF(7)*U*R+KCOEFF(9)*V*Q+KCOEFF(10)*W*P+KCOEFF(11)*W*R FUNC0072
```

```
     2))+((1/L**2)*(KCOEFF(12)*(U**2)+KCOEFF(13)*V*U+KCOEFF(14)*V*DABS(V
     3W)+KCOEFF(15)*V*W+KCOEFF(16)*(U**2)*DELR+KCOEFF(17)*(U**2)*(ETA-1)
     4))+((2/(RHO*L**5))*((YG*WT-YB*B)*DCOS(THETAR)*DCOS(PHIR)-(ZG*WT-ZB
     5*B)*DCOS(THETAR)*DSIN(PHIR)))-((IZ-IY)*Q*R)+(IXZ*Q*P)-((R**2-Q**2)
     6*IYZ)-IXY*(P*R-QDT)-((M*(1/L**2))*(YG*(V*P-U*Q)-ZG*(U*R-W*P)))
C
      RETURN
      END
```

```
C
      SUBROUTINE RUDDER
C
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(
     13),BI(3),CI(3),XCOEFF(16)
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ
      COMMON  T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG
      COMMON    XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG,
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,DELB,THETA,PH
     2I,A1,A2
      COMMON F1,F2,F3,F4,WT,B
      COMMON DELR,RHO,TR,RUD1,U3
      COMMON I1,I2
C
C     TEST FOR WHERE IN THE TURN THE VEHICLE IS AND CHANGE THE AMOUNT OF
C     RUDDER AS APPRCPRIATE.
C
      R1=R
      T22=T2
      IF(R.EQ.0.0) R1=1.0D-20
      APSI=DABS(PSI)
      AR1=DABS(R1)
      TT=(COURSE-APSI)/AR1
      T3=T2*2.0
      DELR1=DABS(DELR)
      IF(I2.GT.0) GC TO 3
      IF(I1.GT.0) GC TO 2
      IF(TT.LE.T3) GO TO 2
      IF(TR.GT.0.0D+1) T22=TR
      IF(T.LT.T22) GO TO 1
      DELR1=DABS(RUDAMT)
      GO TO 5
    1 DELR1=DELR1+DELT*RRATE
    5 DELR=DSIGN(DELR1,RUDAMT)
      RETURN
```

```
    2 I1=1                                                              RUD10037
      DELR1=DELR1-DELT*RRATE                                            RUD10038
      IF(DELR1.LE.0.0) GO TO 3                                          RUD10039
      GC TO 5                                                           RUD10040
    3 I2=1                                                              RUD10041
      DELR=0.0D+1                                                       RUD10042
      RETURN                                                            RUD10043
      END                                                               RUD10044
```

```
C                                                                       RUD20001
      SUBROUTINE ALTRUD                                                 RUD200C2
C                                                                       RUD20003
      IMPLICIT REAL*8(A-H),REAL*8(J-Z)                                  RUD20004
      REAL*8    YCOEFF(22),NCOEFF(22),KCOEFF(17),A(4),E(4),C(4),D(4),AI(RUD20005
     13),BI(3),CI(3),XCOEFF(16)                                         RUD20006
      REAL*8 IX,IY,IZ,IXY,IXZ,IYZ                                       RUD20007
      COMMON  T,DELT,RUDAMT,RRATE,T2,COURSE,PSI,TLAG                    RUD20008
      COMMON    XCOEFF,YCOEFF,NCOEFF,KCOEFF,U,V,W,P,Q,R,QDT,AI,BI,CI,XG,RUD20009
     1YG,ZG,XB,YB,ZB,BIGU,IX,IY,IZ,IXY,IXZ,IYZ,M,L,UO,DELS,DELB,THETA,PHRUD20010
     2I,A1,A2                                                           RUD20011
      COMMON F1,F2,F3,F4,WT,B                                           RUD20012
      COMMON DELR,RHO,TR,RUD1,U3                                        RUD20013
      COMMON I1,I2                                                      RUD20014
C                                                                       RUD20015
C     THIS SUBROUTINE MAINTAINS THE RUDDER AMOUNT AT LESS THAN          RUD20016
C      MAXIMUM ORDERED RUDDER UNTIL U IS LESS THAN 1/3 UO. ALTRUD       RUD20017
C     SHOULD NOT BE USED FOR COURSE CHANGES OF LESS THAN ABOUT 90 DEGS. RUD20018
C                                                                       RUD20019
      R1=R                                                              RUD20020
      IF(R.EQ.0.0) R1=1.0D-20                                           RUD20021
      T23=(DABS(RUD1)/RRATE)+TLAG                                       RUD20022
      APSI=DABS(PSI)                                                    RUD20023
      AR1=DABS(R1)                                                      RUD20024
      TT=(COURSE-APSI)/AR1                                              RUD20025
      T3=T2*2.0                                                         RUD20026
      DELR1=DABS(DELR)                                                  RUD20027
      TR=T2+T-T23                                                       RUD20028
      IF(I2.GT.0) GC TO 3                                               RUD20029
      IF(I1.GT.0) GO TC 2                                               RUD20030
      IF(TT.LE.T3) GC TO 2                                              RUD20031
      IF(T.LT.T23) GC TO 1                                              RUD20032
      DELR1=DABS(RUD1)                                                  RUD20033
      GC TO 5                                                           RUD20034
    1 DELR1=DELR1+DELT*RRATE                                            PUD20035
    5 DELR=DSIGN(DELR1,RUDAMT)                                          RUD20036
```

```
      RETURN                                                            RUD20037
    2 I1=1                                                              RUD20038
      DELR1=DELR1-DELT*RRATE                                            RUD20039
      IF(DELR1.LE.0.0) GO TO 3                                          RUD20040
      GO TO 5                                                           RUD20041
    3 I2=1                                                              RUD20042
      DELR=0.0D+1                                                       RUD20043
      RETURN                                                            RUD20044
      END                                                               RUD20045
```

A.3 SAMPLE OUTPUT

OUTPUT FOR SUBMARINE E - NO MODS

UO =0.230D+02 RUDDER RATE =0.400D+01 COURSE CHANGE =0.360D+03

AMOUNT OF RUDDER =0.350D+02 DELTA T =0.500D+00

T (SEC)	U (KTS)	V (KTS)	R (DEG/SEC)	PHI (DEG)	UDT (KTS/HRS)	VDT (KTS/HRS)	RDT (DEG/SEC/SEC)	PDT (DEG/SEC/SEC)	UELR (DEG)	CRS (DEG)	X (YDS)	Y (YDS)
0.0	0.230D+02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.150D+01	0.230D+02	0.0	0.0	0.0	-.822D-03	0.344D-01	-.876D-01	0.133D+00	0.200D+01	0.0	0.647D+01	0.0
0.200D+01	0.230D+02	0.172D-01	-.438D-01	0.166D-01	-.331D-02	0.718D-01	-.172D+00	0.220D+00	0.400D+01	-.109D-01	0.129D+02	-.608D-02
0.250D+01	0.230D+02	0.531D-01	-.130D+00	0.774D-01	-.762D-02	0.112D+00	-.252D+00	0.265D+00	0.600D+01	-.543D-01	0.194D+02	-.277D-01
0.300D+01	0.230D+02	0.109D+00	-.255D+00	0.199D+00	-.141D-01	0.154D+00	-.327D+00	0.273D+00	0.800D+01	-.151D+00	0.259D+02	-.749D-01
0.350D+01	0.230D+02	0.186D+00	-.419D+00	0.387D+00	-.231D-01	0.199D+00	-.398D+00	0.250D+00	0.100D+02	-.319D+00	0.324D+02	-.163D+00
0.400D+01	0.230D+02	0.286D+00	-.618D+00	0.641D+00	-.352D-01	0.245D+00	-.463D+00	0.200D+00	0.120D+02	-.578D+00	0.388D+02	-.309D+00
0.450D+01	0.230D+02	0.409D+00	-.849D+00	0.952D+00	-.513D-01	0.293D+00	-.522D+00	0.127D+00	0.140D+02	-.945D+00	0.453D+02	-.531D+00
0.500D+01	0.229D+02	0.555D+00	-.111D+01	0.130D+01	-.719D-01	0.340D+00	-.575D+00	0.357D-01	0.160D+02	-.143D+01	0.517D+02	-.848D+00
0.550D+01	0.229D+02	0.725D+00	-.140D+01	0.167D+01	-.980D-01	0.386D+00	-.620D+00	-.707D-01	0.180D+02	-.206D+01	0.582D+02	-.128D+01
0.600D+01	0.228D+02	0.918D+00	-.171D+01	0.204D+01	-.130D+00	0.430D+00	-.658D+00	-.188D+00	0.200D+02	-.284D+01	0.646D+02	-.186D+01
0.650D+01	0.228D+02	0.113D+01	-.204D+01	0.238D+01	-.170D+00	0.472D+00	-.687D+00	-.314D+00	0.220D+02	-.377D+01	0.710D+02	-.260D+01
0.700D+01	0.227D+02	0.137D+01	-.238D+01	0.265D+01	-.216D+00	0.508D+00	-.706D+00	-.445D+00	0.240D+02	-.488D+01	0.773D+02	-.353D+01
0.750D+01	0.226D+02	0.162D+01	-.273D+01	0.283D+01	-.271D+00	0.540D+00	-.715D+00	-.577D+00	0.260D+02	-.616D+01	0.836D+02	-.466D+01
0.800D+01	0.225D+02	0.189D+01	-.309D+01	0.287D+01	-.333D+00	0.565D+00	-.713D+00	-.706D+00	0.280D+02	-.761D+01	0.898D+02	-.603D+01
0.850D+01	0.223D+02	0.218D+01	-.345D+01	0.276D+01	-.402D+00	0.582D+00	-.700D+00	-.825D+00	0.300D+02	-.925D+01	0.959D+02	-.764D+01
0.900D+01	0.221D+02	0.247D+01	-.380D+01	0.246D+01	-.516D+00	0.638D+00	-.797D+00	-.745D+00	0.350D+02	-.111D+02	0.102D+03	-.957D+01
0.950D+01	0.218D+02	0.279D+01	-.420D+01	0.196D+01	-.579D+00	0.605D+00	-.669D+00	-.105D+01	0.350D+02	-.131D+02	0.108D+03	-.117D+02
0.100D+02	0.215D+02	0.309D+01	-.453D+01	0.124D+01	-.641D+00	0.565D+00	-.540D+00	-.124D+01	0.350D+02	-.152D+02	0.113D+03	-.141D+02

A.4 INPUT/OUTPUT VARIABLES

VARIABLE	MEANING	FORMAT	DIMENSIONS	INPUT/ OUTPUT
AI, BI, CI	Set of constants representing the propellar thrust in the X-equation	D10.4	ND	I
A1, A2	Limits used for selecting the proper propeller thrust	D10.4	ND	I
B	Ship's buoyancy	D10.4	lbs	I
COURSE	Amount of ordered course change	D10.4	deg	I, 0
DELB	Initial sail-plane deflection	D10.4	deg	I
DELGM	Amount the original GM is to be changed	D10.4	ft	I
DELRO	Rudder deflection at time T	D9.3	deg	0

[1] ND = non-dimensional

DELS	Initial sternplane deflection	D10.4	deg	I
DELT	Time increment used in iteration	D10.4	sec	I, O
ID	40 character alpha-numeric heading	A1	ND	I, O
INDEX	If >0, read new submarine coefficients. If ≤ 0, run same submarine for new initial conditions.	I2	ND	I
IX, IY, IZ IXY, IXZ, IYZ	Moments of inertia	D10.4	ND	I
KCOEFF	K-equation coefficients	D10.4	ND	I
L	Ship's overall length	D10.4	ft	I
M	Ship's mass	D10.4	ND	I
NCOEFF	N-equation coefficients	D10.4	ND	I

P, Q, R	Initial angular velocity about the x, y and z axes, respectively	D10.4	deg/sec	I
PDT, QDT, RDT	Initial angular accelerations about the x, y and z axes, respectively	D10.4	deg/sec^2	I
PDTO, RDTO	Angular accelerations about the x and z axes, respectively	D9.3	deg/sec^2	O
PHI	Initial angle of roll	D10.4	deg	I
PHIO	Roll angle at time T	D9.3	deg	O
PSI	Initial angle of yaw	D10.4	deg	I
PSIO	Yaw angle at time T	D9.3	deg	O
RHO	Sea water density	D10.4	$slugs/ft^3$	I
RO	Angular velocity about the z axis at time T	D9.3	deg/sec	O

RRATE	Average rudder rate	D10.4	deg/sec	I, 0
RUDAMT	Maximum ordered rudder deflection	D10.4	deg	I,0
RUD1	Value of reduced rudder deflection associated with sub-routine ALTRUD	D10.4	deg	I, 0
T	Present time	D9.3	sec	0
THETA	Initial angle of pitch	D10.4	deg	I
TLAG	Time lag of of the control system	D10.4	sec	I
U, V, W	Initial forward, lateral and vertical velocities	D10.4	kts	I
UDT, VDT, WDT	Initial forward, lateral and vertical accelerations	D10.4	kts/hr	I

UDTO, VDTO	Forward and lateral acceleration, respectively at time T	D9.3	kts/hr	O
UO	Initial forward velocity	D9.3	kts	O
UOO, VO	Forward and lateral velocities, respectively at time T	D9.3	kts	O
WT	Ship's weight	D10.4	lbs	I
XB, YB, ZB	The x, y, z position of the center of buoyancy	D10.4	ft	I
XCOEFF	X-equation coefficients	D10.4	ND	I
XG, YG, ZG	The x, y, z position of the center of gravity	D10.4	ft	I
XX, YY	The moving x and y coordinates of the submarine's origin with respect to a fixed coordinate system, whose origin is the point where the turn begins	D9.3	yds	O

YCOEFF	Y-equation coefficients	D10.4	ND	I

APPENDIX B

B.1 AXIAL FORCE

$$m\left[\dot{u} - vr + wq - x_G\,(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})\right] =$$

$$+ \frac{\rho}{2}\,\ell^4\left[X_{qq}'q^2 + X_{rr}'r^2 + X_{rp}'rp\right]$$

$$+ \frac{\rho}{2}\,\ell^3\left[X_{\dot{u}}'\dot{u} + X_{vr}'vr + X_{wq}'wq\right]$$

$$+ \frac{\rho}{2}\,\ell^2\left[X_{uu}'u^2 + X_{vv}'v^2 + X_{ww}'w^2\right]$$

$$+ \frac{\rho}{2}\,\ell^2u^2\left[X_{\delta r\,\delta r}'\delta r^2 + X_{\delta s\,\delta s}'\delta s^2 - X_{\delta b\,\delta b}'\delta b^2\right]$$

$$+ \frac{\rho}{2}\,\ell^2\left[a_i u^2 + b_i u u_c + c_i u_c{}^2\right]$$

$$- (W - B)\sin\theta$$

$$+ \frac{\rho}{2}\,\ell^2\left[X_{vv\eta}'v^2 + X_{ww\eta}'w^2 + X_{\delta r\delta r\eta}'\delta_r{}^2u^2\right.$$

$$\left. + X_{\delta s\delta s}'\delta_s{}^2u^2\right](\eta - 1)$$

B.2 LATERAL FORCE

$$m\left[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(qp + \dot{r})\right] =$$

$$+ \frac{\rho}{2}\ell^4\left[Y_{\dot{r}}'\dot{r} + Y_{\dot{p}}'\dot{p} + Y_{p|p|}'p|p| + Y_{pq}'pq + Y_{qr}'qr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Y_{\dot{v}}'\dot{v} + Y_{vq}'vq + Y_{wp}'wp + Y_{wr}'wr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[Y_r'ur + Y_p'up + Y_{|r|\delta r}'u|r|\delta_r + Y_{v|r|}'\frac{v}{|v|}|(v^2 - w^2)^{\frac{1}{2}}||r|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_*'u^2 + Y_v'uv + Y_{v|v|}'v|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+ \frac{\rho}{2}\ell^2\left[Y_{vw}'vw + Y_{\delta r}'u^2\delta r\right]$$

$$+ (W - B)\cos\theta\sin\varphi$$

$$+ \frac{\rho}{2}\ell^3 Y_{r\eta}'ur(\eta - 1)$$

$$+ \frac{\rho}{2}\ell^2\left[Y_{v\eta}'uv + Y_{v|v|\eta}'v|(v^2 + w^2)^{\frac{1}{2}}| + Y_{\delta r\eta}'\delta_r u^2\right](\eta - 1)$$

B.3 ROLLING MOMENT

$$I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - q)I_{xy}$$

$$+ m\left[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)\right] =$$

$$+ \frac{\rho}{2}\ell^5\left[K_{\dot{p}}'\dot{p} + K_{\dot{r}}'\dot{r} + K_{qr}'qr + K_{pq}'pq + K_{p|p|}'p|p|\right]$$

$$+ \frac{\rho}{2}\ell^4\left[K_p'up + K_r'ur + K_{\dot{v}}'\dot{v}\right]$$

$$+ \frac{\rho}{2}\ell^4\left[K_{vq}'vq + K_{wp}'wp + K_{wr}'wr\right]$$

$$+ \frac{\rho}{2}\ell^3\left[K_*'u^2 + K_v'uv + K_{v|v|}'v|(v^2 + w^2)^{\frac{1}{2}}|\right]$$

$$+ \frac{\rho}{2}\ell^3\left[K_{vw}'vw + K_{\delta r}'u^2\delta r\right]$$

$$+ (y_GW - y_BB)\cos\theta\cos\varphi - (z_GW - z_BB)\cos\theta\sin\varphi$$

$$+ \frac{\rho}{2}\ell^3 K_{*\eta}'u^2(\eta - 1)$$

B.4 YAWING MOMENT

$$I_z\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx}$$

$$+ m\left[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)\right] =$$

$$+ \frac{\rho}{2}\ell^5\left[N_{\dot{r}}'\dot{r} + N_{\dot{p}}'\dot{p} + N_{pq}'pq + N_{qr}'qr + N_{r|r|}'r|r|\right]$$

$$+ \frac{\rho}{2}\ell^4\left[N_{\dot{v}}'\dot{v} + N_{wr}'wr + N_{wp}'wp + N_{vq}'vq\right]$$

$$+ \frac{\rho}{2}\ell^4\left[N_p'up + N_r'ur + N_{|r|\delta r}'u|r|\delta r + N_{|v|r}'\left|(v^2 + w^2)^{\frac{1}{2}}\right|r\right]$$

$$+ \frac{\rho}{2}\ell^3\left[N_*'u^2 + N_v'uv + N_{v|v|}'v\left|(v^2 + w^2)^{\frac{1}{2}}\right|\right]$$

$$+ \frac{\rho}{2}\ell^3\left[N_{vw}'vw + N_{\delta r}'u^2\delta r\right]$$

$$+ (x_G W - x_B B)\cos\theta\sin\varphi + (y_G W - y_B B)\sin\theta$$

$$+ \frac{\rho}{2}\ell^4 N_{r\eta}'ur(\eta - 1)$$

$$+ \frac{\rho}{2}\ell^3\left[N_{v\eta}'uv + N_{v|v|\eta}'v\left|(v^2 + w^2)^{\frac{1}{2}}\right| + N_{\delta r\eta}'\delta_r u^2\right](\eta - 1)$$

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